SUPPLEMENT

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IN presenting to the Public the completion of the College MATHEMATICAL EXAMINATION PAPERS up to the present time, the Publisher desires respectfully to return his sincere thanks to those Gentlemen who so kindly allowed him the use of their Papers.

Some Papers of a date previous to the year 1820 have been inserted, contrary to the original design, on account of their value and scarcity, several of them having been set by SIR J. W. F. HERSCHELL.

Cambridge, April 1833.

CONTENTS.

MECHANICS.

TRINITY COLLEGE, 1818.

1. DISTINGUISH between the real and apparent motions, of a person walking across the deck of a ship under sail; and also of a heavy body let fall from the mast-head:—and, for each case, prove their agreement geometrically.

2. If the angle, at which two given forces act, be diminished, the compound force is increased.

3. If a body be kept at rest by three forces, and lines be drawn, making each the same angle with the directions in which they act, and towards the same parts,—the sides of a triangle formed by these lines will represent the quantities of the respective forces.

4. Enumerate the simple mechanical powers; and give familiar instances of the lever (of both kinds), the wheel and axle, and the wedge.

5. Prove the general proposition of the screw: and find numerically the weight that could be sustained by a power of 1 lb. at the distance of three yards from the axis of the screw,—the distance of two contiguous threads being one inch.

6. If a lever be put in motion, the velocity of power : velocity of weight :: weight : power.

7. A spherical body rests upon two planes, inclined to the horizon at the angles of 45° and 60° respectively. Compare the pressures.

8. Investigate the rule for finding the centre of gravity of any number of particles of matter A, B, C, D, &c.; and demonstrate geometrically, that the same point will be discovered, in whatever order the particles are taken, (e. g. whether we find E the centre of A and B, then the centre of C and $A + B$ placed at E, and so on,—or whether we begin with finding the centre of A, C, &c. &c.)

9. If a triangle, whose sides are in the ratio of 3, 4, 5, be suspended by the centre of the inscribed circle,—shew that it cannot remain at rest, unless the shorter side be in an horizontal position.

10. Find the centre of gravity of a quadrilateral pyramid.

11. Prove that the relative velocity before impact : relative velocity after :: force of compression : force of elasticity:—and that in all cases of imperfect elasticity, this ratio is greater than that of $Aa^2 + Bb^2$ to $Ap^2 + Bq^2$, (where a and b are the velocities of A, B before impact, p and q their velocities after.)

12. Shew that the velocity communicated by A to C through B, where B is of an intermediate magnitude, is greater than what would be immediately communicated from A to C in the case of perfect elasticity;—also that this velocity is the greatest, when B is a mean proportional between A and B.

13. If a body strike upon a plane at an angle of incidence θ, and with velocity v—prove that the direction and velocity, after reflection, are found by the proportions $\tan.\theta' : \tan.\theta ::$ force of compression : force of elasticity, and $v' : v :: \sin.\theta : \sin.\theta'$. What must be the angles of incidence and reflection—where the velocity before impact : velocity after :: $\sqrt{2} : 1$, and force of compression : force of elasticity :: $\sqrt{3} : 1$? And how do the general propositions apply to the case of perfectly hard bodies?

14. If v, v' be any two velocities of a body either falling or rolling down an inclined plane—what will be its velocity at the middle point of space between them?

15. A body (P) weighing 1 lb. is thrown downwards from the top of a tower 270 feet high, with a velocity of 40 feet in a second; and at the same instant a body (Q) weighing 9 lbs. is thrown upwards in the same line from the foot of the tower with a velocity of 50 feet in a second. In what time and at what height will they meet? and after the impact (supposing them perfectly elastic) what will be their velocities and directions; and how long will it be before each of them reaches the ground?

16. Given a point without a circle; it is required to find the line of quickest descent to the circumference.

17. If on an inclined plane, of which the length $= a$, and height $= b$, a part be taken equal to b, and described in an equal time by a body whose descent began from the top; what is the distance of either extremity of this part from the vertex?

18. If a body begin to oscillate from the highest point of an inverted cycloid,—shew that the time of describing any given arc of the cycloid, is measured by the arc of the generating circle intercepted between the ordinates to the axis, drawn from the extremities of the arc. Compare also the time of its describing any arc of the cycloid measured from the summit, with the time of descent down the chord.

19. If a clock gain or lose t seconds in a given time $(= T'')$, then if the error be small, prove that the quantity by which the length (l) of the pendulum is to be corrected, will be $\frac{2lt}{T}$.

20. How far below the earth's surface, or how far above it, must a pendulum, shorter than 39·2 inches by a very small fraction (x), be taken to vibrate seconds? Earth's radius $= r$.

21. If a body P be connected by a pulley with a weight W on a plane inclined to the horizon at a given angle I;—required the space described by the force of gravity in t'', and also the velocity it will acquire.

22. Prove that the elevation at which a projectile must be thrown from a plane of given inclination (I), in order that the range may be n times the space due to the velocity of projection,—is determined by the two values of E in the expression

$$\sin.(I + 2E) = \sin.I + \frac{n}{2}.\cos.^2 I.$$

Compare the terms of flight, and the greatest heights, for the two values of E thus found, where the plane is horizontal, and $n = 1$; and also where $I = 30^\circ$, and $n = \frac{4}{3}.(\sqrt{3}-1)$.

23. A body is projected from a given height with a given velocity. Required the locus of all the greatest heights, for every direction in which it may be thrown.

24. A hollow paraboloid being placed with its vertex downwards, and terminated above by an horizontal circular plane perpendicular to the axis,—it is required to find by construction a point in any given diameter of the circle, such that a body let fall from it on the surface of the paraboloid, will, after one rebound, hit the vertex.

TRINITY COLLEGE, 1819.

1. DETERMINE the quantity and direction of the compound force resulting from any number of given forces acting on a point.

1st, When the forces are in the same plane.

2nd, When they are in different planes.

2. AB is an uniform lever, of given length and weight, moving freely about A. A string is fixed at B distended by a weight passing over a fixed pulley E, placed at the horizontal distance $AE = AB$. Determine the position in which the lever will rest.

3. In a system of pullies, where each hangs by a separate string, and each string is fixed to the weight; supposing the power and weight in motion,

1st, Prove that velocity of P : velocity of W :: W : P.

2nd, Compare the velocities with which the different pullies revolve round their axes.

4. If a plane pass through the centre of gravity of a system of bodies, prove that the sum of all the products formed by multiplying each of the bodies into its perpendicular distance from the plane on one side, is equal to the sum similarly taken on the other side of the plane.

5. Determine the proportion of the sides of a right-angled triangle, so that the time down the length may equal the time down the height + time of describing the base with the last acquired velocity continued uniform.

6. Prove that if the same triangle be suspended by the centre of the inscribed circle, it will only rest when the shortest side is parallel to the horizon.

7. If two bodies balance each other on two inclined planes by means of a string passing over their intersection, prove that when the bodies are put in motion their centre of gravity will neither ascend nor descend.

8. Determine the sides of the strongest rectangular beam which can be cut out of a given cylindrical piece of wood.

9. If a body be moved from rest through the space S, by the action of an uniform force during T'', and acquire the velocity V; prove that $S = \frac{T \times V}{2}$.

10. Having established the preceding equation, deduce the two following:

$$M = Q \times V^2 \times \frac{1}{4mS}$$

$$M = Q \times V \times \frac{1}{2mT}$$

Where M is the moving force, and Q the quantity of matter.

11. A body projected up an inclined plane where $H = \frac{L}{8}$ describes 32 feet in 2″. How far will it ascend before all its velocity is lost?

12. A body, whose elasticity $= \frac{1}{2}$, descends from the height of 10 feet, and rebounds till all its velocity is lost; required the whole time of its motion, and the whole space passed over.

13. If in a common cycloid an ordinate be drawn from the point which bisects the axis, and from the extremity of that ordinate another line be drawn to the vertex; prove that the cycloidal segment thus cut off $= \frac{1}{8}$ of the square upon the axis.

14. Given the force of gravity; determine the time in which a body will vibrate in a cycloid whose axis $= A$.

15. A pendulum of unknown length is observed to vibrate 59 times in a minute; it is then shortened three inches, and is observed to vibrate 61 times in a minute. What is the length of a pendulum vibrating seconds in the same latitude?

16. When a body vibrates in the complete arc of a cycloid:

(1). Compare the tension of the string in every point of the curve arising from gravity, with the tension arising from centrifugal force.

(2). Determine the point where the accelerating force down the curve = the tension of the string.

17. If a body oscillate in the complete arc of a cycloid; prove that the time of its descent to any point, is measured by the arc of the circle upon the axis cut off by the ordinate to that point.

18. Given the point of projection and the velocity,

(1). Determine the direction, so that the ball may strike a given point in the under surface of a given horizontal plane above the point of projection.

(2). Under the same circumstances, determine the direction so that the range may be a maximum.

19. If grape shot be discharged from a cannon; prove that at the end of any given time they will be found in the surface of a given sphere.

20. A heavy beam AB has one end (A) fixed in the ground, and is supported by a prop CD of given length. If the angle A be given, determine the position of CD so that it may sustain the least possible pressure.

TRINITY COLLEGE, 1820.

1. What is the proportion between the power and the weight, in the system of pullies where each pulley hangs by a separate string?

2. A weight P upon an inclined plane is supported by a weight Q hanging freely, the string being parallel to the plane. Shew that if they be moved into any other position, the centre of gravity moves in an horizontal line.

3. If P and Q, in the last question, be not in equilibrium, what will be the path of the centre of gravity?

4. Explain and prove the second law of motion.

5. In the collision of elastic bodies, the relative velocity after impact, is to that before, as the imperfect to perfect elasticity.

6. A given weight P, by means of a string passing over a fixed pulley, draws up a chain, which was previously coiled upon an horizontal plane directly below the pulley; find the velocity acquired by P in descending through a given space.

TRINITY COLLEGE, 1820.

Statics.

1. When forces keep each other in equilibrium round a fixed point, the sum of all their moments is $= 0$; those being reckoned negative which tend to turn the system in the opposite direction.

2. Find the resultant of any number of forces in the same plane acting on a point. Apply the formulæ to the following example:

   AB, AC, AD are three lines making angles of 120° with each other; the point A is acted on by pulling forces in AB and AC which are as 3 and 4, and by a pushing force in DA which is as 5. Find the force which will keep it at rest.

3. A string fastened at A and passing over a fixed pulley B, has a known weight W hung by a knot at C; find what weight must be appended at B, that CB may be horizontal.

4. A weight Q hanging freely, supports an equal weight P upon an inclined plane, by means of a string passing over a pulley below the plane: find the position of equilibrium.

5. When a body is sustained upon a curve whose co-ordinates are x and y, by any forces whose components in those directions are X and Y, shew that $Xdx + Ydy = 0$. Apply the formula to find the position of equilibrium when a weight Q hanging freely, supports a weight P upon a parabola whose axis is horizontal, by means of a string passing over the focus.

6. Find the centre of gravity of any number of points in the same plane.

7. The sum of the squares of the distances of three equal bodies from each other, is three times the sum of the squares of their distances from their common centre of gravity.

8. Prove the differential expression for the centre of gravity of any solid of revolution; and find the centre of gravity of a hemisphere.

9. *ABCD* is a quadrilateral figure of which the two shorter sides *AB, BC* are equal, as also the two longer *AD, DC*; and the angle *ABC* is a right angle: what is the greatest length of the side *AD* that the figure may stand on the base *AB* on an horizontal plane without oversetting?

10. Given a bent lever with arms of uniform thickness, moveable in a vertical plane about the angular point: find the position in which it will rest.

11. A given beam considered as a line is supported on two given inclined planes: find the position of equilibrium.

12. Given the pressure upon one of the four legs of a rectangular table of known weight; find the pressures of the other three. Shew that without this datum the problem is indeterminate.

13. *ABC* is a right-angled isosceles triangle, and three equal forces act in the lines *AB, BC, CA*. At what point of the plane *ABC*, produced if necessary, must a force be applied to keep it at rest, and what must be its magnitude and direction?

14. A beam *BC* hangs by a string *AB* from a fixed point *A*, with its lower extremity *C* upon an horizontal plane: find the position in which it will rest. Also find the horizontal force which must be applied at *C* to retain it in a given position.

15. A false balance has one of its arms exceeding the other by $\frac{1}{m}$ of the shorter. It is used, the weight being put as often in one scale as the other. What is the shopkeeper's gain or loss *per cent.*?

16. In an arch which is in equilibrium, the weights of the voussoirs are as the differences of the tangents of the angles which their joints make with the vertical.

Dynamics.

1. A bow is drawn by a force of 50 lbs.; the weight of the arrow being $\frac{1}{10}$ lb., compare the force of gravity with the initial accelerating force which the string exerts upon the arrow, when it is let go; neglecting the inertia of the bow.

2. If a, b, be the velocities of two bodies A, B before their direct impact; u, v the velocities after, α and β the velocities gained and lost respectively, and e the fraction which measures the elasticity;

$$Aa^2 + Bb^2 = Au^2 + Bv^2 + \frac{1-e}{1+e}(A\alpha^2 + B\beta^2)$$

3. A and B are two given points in the diameter of a circle: find in what direction a perfectly elastic body must be projected from A, so that after reflection at the circle it may strike B.

4. Prove that if a body be accelerated by a constant force

$$v = ft \text{ and } s = \tfrac{1}{2}ft^2.$$

5. Find the velocity and direction with which a body must be projected from a given point that it may hit two other given points in the same vertical plane.

6. AB is the vertical diameter of a circle: a perfectly elastic body descends down the chord AC; and being reflected by the plane BC, describes its path as a projectile. Shew that this path strikes the circle at the opposite extremity of the diameter CD.

7. Find the equation to the *cycloid;* and shew that in the same cycloid the oscillations are *isochronous*.

TRINITY COLLEGE, 1821.

1. If forces proportional to the sides of a quadrilateral figure, be applied perpendicularly at their middle points, they will keep one another in equilibrium.

2. The moment of the resultant of any forces is the sum of the moments of the components.

3. At what point of a vertical pillar must a rope of given length be fixed, so that a man pulling at the other end may exert the greatest force in upsetting it?

4. If AP, BQ represent two forces acting on the equal arms AC, CB of a lever whose fulcrum is C, also if AP, BQ make given angles with the horizon, and Pp, Qq be perpendicular to ACB, shew that there will be an equilibrium when $Ap + Bq$ is a maximum.

5. A given weight W is supported by n strings passing over pullies, placed at the angles of a regular polygon, whose plane is horizontal, each string being fastened to an equal weight P: find the position in which W will rest.

6. One sphere is supported by three others which touch, find the pressures on each; also the horizontal pressures necessary to prevent them from sliding.

7. A given weight, suspended by a string of given length, is drawn horizontally by a given force, find the position into which it will be drawn.

8. Investigate the expression for the centre of gravity of an area, and apply it to the quadrant of an ellipse.

9. The sum of the squares of the distances of the centre of gravity of any number of equal bodies from the centre of gravity of each, is equal to the sum of the squares of the distances of the centres of gravity of these bodies, taken two and two, divided by the number of bodies.

10. If any system of bodies, acted on by gravity alone, be in equilibrium, its centre of gravity is either the highest or lowest possible.

11. AC and BD are two uniform beams, moveable in a vertical plane about the fixed points A and B, also AC is equal to AB; required the position in which the beams will support each other.

12. Find the equation between P and W, when the weights of the pulleys are equal, and each hangs by a separate string.

13. If two hemispheres rest, with the convex surface of one placed on that of the other, shew that the equilibrium will be stable, or unstable, according as the radius of the upper one is less, or greater than three-fifths of the radius of the under.

14. A and B are two equal balls at rest; required their position, so that if a perfectly elastic ball C, impinge upon them in a direction perpendicular to, and bisecting the line joining their centres, the relative velocities of A and B after impact may be the greatest possible.

15. Draw *geometrically* the line of quickest descent from the focus of a parabola to the curve, the axis being vertical, and the vertex uppermost.

16. The space in any time is equal to the space described with the velocity of projection, plus, or minus the space described from rest by the action of the force, according as the body is projected in the direction of the force, or contrary to it.

17. Two balls A and B, of which B is perfectly elastic, are let fall at the same instant from two given points in the same vertical line: find the point where B, after rebounding from the horizontal plane, will meet A.

18. A chain of given length has part on a table, and part hanging over it; find the time in which it will fall off the table.

19. If a spiral tube wind round the surface of a paraboloid, standing on an horizontal plane, and make a given angle with the generating parabola, find the accelerating force on a body descending in the tube; and prove that if it descends from a point very near the vertex, it will make its successive revolutions in equal times.

20. Find the equation to the curve described by the centre of gravity of two bodies, projected with given velocities, and in given directions in the same vertical plane.

21. If r be the range, and t the time of flight of an inelastic ball on an horizontal plane, then if the same ball had an elasticity e, $\frac{r}{1-e}$ would be the whole space described by it, and $\frac{t}{1-e}$ its whole time of motion.

22. A pendulum which vibrates seconds at Greenwich, taken to another place on the earth's surface, loses n seconds a day; compare the force of gravity at the two places.

23. The time of descent down any arc of a cycloid, from the highest point, is less than the time down the chord.

24. If A be the lowest point of a circular arc AB, and if AI be taken equal to $\frac{AB}{2}$, and the chord AO be taken to the chord $AI :: \sqrt{2} : 1$, prove that the time down BO is equal to the time down the remaining arc OA.

TRINITY COLLEGE, 1822.

1. If any two forces, acting at a point, be represented in magnitude and direction by the sides of a parallelogram, the resultant is represented in magnitude and direction by its diagonal.

2. Prove, that, if several forces keep each other in equilibrium round a point, each is equal and opposite to the resultant of all the others.

3. If α, β, γ and α', β', γ' be the inclinations of two lines to three rectangular axes drawn through their point of intersection; and a force F act along one of the lines: prove that its value estimated along the other is $F.(\cos.\alpha.\cos.\alpha' + \cos.\beta.\cos.\beta' + \cos.\gamma.\cos.\gamma')$.

4. The resultant of two parallel forces is parallel to them, equal to their sum, and its direction divides the line which joins their points of application in the inverse ratio of the forces. Construct for the resultant when the forces do not act towards the same parts.

5. Two weights are suspended from knots in a string the ends of which are fastened to two fixed points: given one of the weights, and the lengths of the suspending strings between the knots and the points where the other string produced would meet them.—Find the other weight.

6. Determine the distances of the centres of gravity of a right cone, and of a spherical surface from the vertex of each; and hence deduce the distance of the centre of gravity of a spherical sector from the centre of the sphere.

7. A given heavy spherical bowl is loaded at a certain point of its edge with a given weight: find the position in which it will rest on an horizontal plane.

8. A right cone of given dimensions and weight stands on its vertex with its axis at a given inclination to the horizon: find the magnitude and direction of the least force, acting at the centre of its base, which will keep it in that position.

9. Find the ratio of the power to the resistance in the wheel and axle, without supposing them to act in the same plane.

10. Find, generally, the ratio of the power to the resistance in the toothed wheel; and also, when the teeth are small: and prove the principle of virtual velocities on the latter supposition in this mechanical power.

11. In an arch which is in equilibrium, the weights of the voussoirs are as the differences of the tangents of the angles which their joints make with the vertical.

12. A ball of given elasticity falls from a given height upon a hard plane: determine the whole time before the cessation of the motion.

13. The straight line of quickest descent from a point within a circle to its circumference passes through its highest point, and that from a point without the circle to its circumference passes through its lowest point.

14. A ball having descended to the lowest point of a circle through an arc whose chord is C drives an equal ball up an arc whose chord is c: shew that the common elasticity (e) of the two balls may be found from this proportion $1 : e :: C : 2c - C$.

15. A body is projected in a given direction, and with a given velocity, and is acted on by the constant force of gravity in parallel lines: find the equation to its path; shew that it is a parabola, and construct it.

16. Assuming that the brachystochronic curve between two points is an arc of an inverted semicycloid, with its base horizontal and the extremity of its base at the higher point: shew how this cycloid may be constructed.

17. Find at what point in the direction of its axis a straight rod of small thickness must be suspended that its oscillation may occupy a given time, and find the lowest limit of that time.

18. If the force to a centre is $\frac{D}{A}$ where D is the distance, A is a given line, and the force of gravity is the unit of force: prove that a cycloidal pendulum, whose length is A, will oscillate in any time,

through the same space as a body drawn by that force from the distance D would move over in that time.

19. A heavy ring R hanging on a thread fastened at A and B, oscillates through a very small arc in the vertical plane of A and B: find the time of an oscillation, neglecting the magnitude of the ring, and the inertia of the string.

20. The uniform triangular plate ABC whose weight is known, is supported by three known weights a, b, c, connected with the angular points, A, B, C, by strings passing through a fixed ring at D: find the lengths AD, BD, CD; and the angles which they make with the vertical.

21. If a tennis ball in rapid motion strikes a vertical wall at a very acute angle, it will describe a curve in the air, so as to return to the wall after having rebounded from it: explain this phenomenon.

22. A given moving force will communicate the same velocity to the centre of gravity, to whatever body in a system it is applied.

TRINITY COLLEGE, 1823.

1. Two Momenta, which when communicated separately, would cause a body to describe the adjacent sides of a parallelogram, will, when communicated together, cause it to describe the diagonal, with an uniform motion.

2. If the angle at which two given forces act is increased, their joint effect is diminished.

3. Two weights will balance each other on an horizontal lever, when they are inversely as their distances from the fulcrum.

4. There are two wheels, whose diameters are 5ft. and 4ft. on the same axle, the diameter of which is 20 inches. What weight on the axle would be sustained by forces equal to 48 lbs. and 50 lbs. on the larger and smaller wheels respectively?

5. Find the relation between P and W, when they sustain each other by a system of pullies, with the same string round all the pullies: on the principle, that if the state of rest be disturbed, P and W will be inversely as their incipient velocities.

6. If two weights sustain each other on two inclined planes, by a string which passes over a pulley at their common vertex, so as to be parallel to the planes respectively; then shall the weights be inversely as the lengths of the planes.

7. Explain the contrivance called a Lewis, for raising heavy stones.

8. Three forces cannot sustain each other on a wedge, unless their directions pass through the same point.

9. The mechanical advantage of the screw, is independent of the radius of the screw.

10. The distance of the common centre of gravity of any number of particles from a given plane, remains the same, however the particles are moved about in planes parallel to the given plane.

11. With what velocity must a ball impinge on another equal ball moving with a given velocity v, that its velocity may be destroyed by the impact; the common elasticity of the balls being $\frac{1}{n}$ th of perfect elasticity?

12. Having given the diameters of two balls moving in the same plane, and the velocities and directions of their motion, find the places of their centres when they come into contact.

13. If a space be described with a velocity uniformly accelerated from rest, it will be the half of the space which would have been described in the same time, had the velocity been uniform and equal to that at the end of the time.

14. The time of an oscillation in a cycloidal arc is constant, whatever be the portion of the whole cycloid: prove this, and find the actual time of an oscillation.

15. The path of a projectile near the surface of the earth is a parabola nearly.

TRINITY COLLEGE, 1823.

1. Express the resultant of two given forces in terms of the components, and of the angle at which they act.

2. Enumerate the different principles on which the preceding question has been resolved.

3. Forces, represented by straight lines drawn from the angular points of a triangle to its centre of gravity, will be in equilibrium.

4. The effects of forces, when estimated in given directions, are not altered by composition or resolution.

5. There is no tension sufficient to keep a heavy cord stretched in any line that is not vertical.

6. A ladder rests against a wall; find its pressure against the wall and on the ground respectively.

7. Define the centre of gravity, and prove that every body has a centre of gravity.

8. If two weights support each other upon any machine, and it be put in motion, the centre of gravity of the weights will, at first, neither ascend nor descend.

9. What is the least slope down which a regular hexagonal prism could roll?

10. A weight slides on a thread fastened to the extremities of equal arms of a lever of uniform density: shew that the lever will not rest except in a vertical or an horizontal position; and that, if it be put in motion, it will ultimately rest in a vertical position.

11. Deduce the differential expression for the centre of gravity of a solid of revolution; and apply it to find the centre of gravity of a cone.

12. An endless cord, passing through rings at given points A, B, on the same level, has equal weights attached to rings at C and D, and a ring at E, equally distant from A and B, supported by a force equal to half of either of those weights: find the angles of the figure which the thread will assume; and the length of the thread, that its tension may be to the supporting force, in the subduplicate ratio of 5 to 4.

13. Apply a given force perpendicularly to a straight lever, so as to keep it at rest, when it is acted on perpendicularly at given points, by two given forces not in the same plane.

14. If a piece of timber 17 feet long, be rested on a prop placed 4 feet from one end, it is found that an hundred weight at that end would be balanced by 12 pounds at the other; and that, if the places of the weights are exchanged, the prop must be 8 feet from the other end. Find the weight of the timber, and the place of the prop on which the timber would balance without the weights.

15. Find the equation of equilibrium on the inclined plane, when friction, proportional to the pressure, is taken into the account.

16. The relative velocity after the direct impact of perfectly elastic bodies is the same as before the impact.

17. Find the direction in which a perfectly elastic billiard-ball must be struck, that, moving from a given point on a five-sided billiard-table, it may, after impinging on the first, third, fifth, second, and fourth sides in order, be reflected to a given point.

18. If two bodies begin to fall at the same time from the common vertex of two inclined planes, the line joining them will move parallel to itself.

19. Required the plane of speediest descent from one given sphere to another.

20. The weight A after descending freely through a feet, begins to draw up another greater weight B by a string passing over a pulley: find the extreme height to which B could rise, and the time of rising.

21. A body is projected from the summit of a mountain of 30° elevation, so as just to strike at the bottom, and with double the velocity of projection, which is what would have been acquired in falling down 400 yards of vertical height. Find the height of the mountain, and the greatest height attained by the projectile.

22. Required the length of an inclined plane, the height of which is one half of its base, that a body projected directly up the plane with a given velocity may be as long after leaving the plane, before it again meets the horizon, as it was in ascending the plane: also find the range and the time of flight.

23. Explain the method of applying a pendulum, so as to regulate the motion of a clock, and the method of producing the requisite angular velocity in the hands.

24. Determine the motion of a given heavy particle, projected along a rod which is supported on a fulcrum at its middle point, and is of a given uniform density; the motion beginning from the middle point, and the first position of the rod being horizontal.

TRINITY COLLEGE, 1824.

1. If the quantities and directions of two forces acting upon a point be represented by two adjacent sides of a parallelogram; its diagonal will represent the quantity and direction of their resultant.

2. Three weights A, B and C are suspended from given points of a straight lever: where must the fulcrum be placed, in order that the lever may be at rest?

3. What is meant by the principle of virtual velocities? Shew its application in the case of equilibrium upon an inclined plane.

4. Find the distance of the centre of gravity of any number of given bodies from a given plane, the distance of the centre of gravity of each body from the plane being given.

5. What are the laws of motion? Can they be considered as entirely founded upon observation and experiment.

6. P (3) draws up Q (5) by means of a string passing over a fixed pulley: find the force accelerating P's descent, and the space described in $t''(10)$: the weight of the string, and the inertia of the pulley being neglected.

7. Find the time of oscillation in a cycloid.

8. If the length of the seconds pendulum be 39·1386 inches, what must be the length of a pendulum which loses 10″ in 24 hours, the force of gravity being diminished by $\frac{1}{1000}$th part of the whole?

9. Find the range and time of flight of a projectile upon an inclined plane passing through the point of projection.

10. In the impact of bodies, whether elastic or not, the velocity of the centre of gravity is the same before and after impact.

TRINITY COLLEGE, 1826.

1. How are forces compared and measured?

2. Find a single force equivalent to two given forces acting at the same point in given directions.

3. Exhibit the algebraical value of the compound force, and show in what cases it is equal to the sum or difference of the given forces.

4. In what case can forces acting at different points be compounded, and how?

5. How may two blocks be formed so as to answer the purpose of several pullies in the system where the same string passes round all?

6. What is the principle of virtual velocities? Prove that it obtains in the case of the double inclined plane.

7. Demonstrate that, when weights balancing each other in all positions on a machine are set in motion, the common centre of gravity remains in the same horizontal plane.

8. Explain the graduation of the Danish steelyard, in which the places of the weights are invariable and the fulcrum is moveable.

9. What is rackwork? Explain the construction and mechanical advantage of the jack, used by masons to lift up large stones.

10. How is the equilibrium of forces acting in different directions on a rigid body stated algebraically?

11. Distinguish between stable and unstable equilibrium. What must be the height of a parabolic conoid resting on its vertex in an equilibrium of indifference?

12. Find the centre of gravity of a wedge, of which the sides are cut into the form of a parabola, the flat surfaces being exactly similar and equal.

13. Find the equation to the catenary measuring the co-ordinates from the lowest point. Show that it nearly coincides with a parabola, about the vertex.

14. Prove the equations of motion:

$$v = \frac{ds}{dt}, \quad f = \frac{d^2s}{dt^2}, \quad vdv = fds.$$

15. Under what conditions may one perfectly elastic ball be made to strike another, so that each shall move, after the impact, in a given direction?

16. How is the velocity, communicated from one mass to another, increased by interposing others between them? What is the utmost extent of the advantage to be obtained in this manner?

17. When any number of bodies not urged by any forces are set in motion, their common centre of gravity moves uniformly in a straight line.

18. Describe the motion of a ball projected up an inclined plane with such a velocity as to fly over the top of it. Show that the parabola which it describes has the same directrix as that in which it would have moved had the inclined plane not existed.

19. Find the time of oscillation of a pendulum, and show how it may be made to oscillate isochronously.

20. How is the intensity of the force of gravity estimated by observations on a pendulum?

21. Find the correction to be applied to a pendulum which vibrates seconds nearly, but not exactly. How is this correction applied by means of a screw?

22. Find the equations to the motion of a point on a curved surface, and apply them to the case of a hollow parabolic conoid, with it axis vertical. Show that when the path becomes a plane circle, the angular velocity is the same in all cases for the same surface.

23. What is D'Alembert's principle? Apply it to determine the motions of two weights connected by a string hanging over a pulley.

24. Explain the construction of Attwood's Machine, and describe the experiments made with it to illustrate the principles of Mechanics.

TRINITY COLLEGE. MAY 1831.

1. If two weights, acting perpendicularly upon a lever, on opposite sides of the fulcrum, have their distances from the fulcrum inversely as the weights, they will balance each other.

2. Shew that, *cæteris paribus*, the larger a carriage-wheel is, the less is the force requisite to draw the carriage over a given obstacle.

3. When three forces acting on a point are in equilibrium, each varies as the sine of the angle contained by the directions of the other two.

4. Required the proportion of P to W in the single moveable pulley, the strings not being parallel.

5. Find the proportion of P to W, when there is equilibrium on the inclined plane, P acting in a direction making any angle with the plane; and shew that this proportion is the same as that of W's velocity in the direction of its action, to P's velocity in the direction of its action, supposing a small motion to be given to the weights.

6. Find the centre of gravity of a pyramid whose base is a triangle, and thence derive that of a cone.

7. State the three laws of motion. What are the quantities to which in mathematical language the terms velocity, momentum, accelerative force, and moving force are applied?

8. Explain the nature of impact, and the manner in which time must be considered in estimating its effect. Illustrate your explanation by examples.

9. The space described by a body uniformly accelerated from rest, is half the space described in the same time with the last acquired velocity.

10. Obtain the equation to the path of a projectile, and the velocity at any point.

11. Find the time of a small oscillation in a circular arc.

12. Give satisfactory reasons for concluding that gravity at the Earth's surface is a constant force, and that it acts on all bodies alike.

TRINITY COLLEGE, May 1831.

1. Find the magnitude and the direction of the resultant of two given forces acting in given directions on a point.

2. A given weight W is to be supported by a horizontal rod of given length l, on two vertical props, one of which can sustain no

more than P, and the other no more than Q; required the point of the rod from which W must be suspended, that the props may support less than the greatest they can support, by the same quantity.

3. Find the proportion of the power to the weight when there is equilibrium on the screw.

4. If a heavy homogeneous triangle be held in any position by three vertical strings attached to its angles, the strings sustain equal portions of it.

5. In the common balance, the weights being unequal, find the position in which it will rest. Hence determine the stability and sensibility of the balance.

6. A flexible chain of given weight is wrapped exactly round a given circle, the plane of which is vertical, and is supported on the circle: required the tensions at the highest and lowest points.

7. A ladder of given weight and dimensions rests in a given position against a vertical wall, and is prevented sliding by friction; having given the ratios of friction to pressure on the horizontal plane and on the wall, find how high a man of given weight may ascend the ladder before it begins to slide.

8. Any number of beams arranged as sides of a polygon, in a vertical plane, support each other, and support also given weights at the angles: it is required to find the horizontal pressure at the points of support.

9. If a couple of equal and opposite forces act in a rigid plane, shew that equilibrium may be produced in an unlimited number of ways by introducing another couple: and assuming arbitrarily a line in which one of the forces of the additional couple shall act, determine the line in which the other must act.

10. Describe the construction of a dome: shew that no dome can remain at rest by its own weight, when supported on a horizontal plane; and that the weights of the consecutive rings of voussoirs may increase in any proportion greater than that of the difference of the tangents of the angles which the joints make with the vertical.

11. State the principle of virtual velocities; and prove by means of it that if a uniform rod rests on two straight lines the equations of

which are $y=ax$, $y=-a'x$, the y's being vertical, the tangent of the angle it makes with the horizon $=\frac{1}{2}\left(\frac{1}{a}-\frac{1}{a'}\right)$.

12. Find the centre of gravity of the frustum of a paraboloid, having given the length c of its axis, and the radii a, b, of its larger and smaller ends.

13. A solid is generated by a variable rectangle moving parallel to itself along an axis perpendicular to its plane through its centre; one side of the rectangle varies as the distance from a fixed point in the axis, half the other is the sine of a circular arc of which this distance is the versed sine: shew that the distance of the centre of gravity of the whole solid from the fixed point, is equal to four-fifths of the length of its axis.

14. A flexible chain of given weight and length hangs vertically; required the horizontal deviation from the vertical line of suspension which its lowest point may be made to undergo by means of a given force, applied horizontally.

15. Assuming that for a constant force $f=\frac{v}{t}$, shew that for a variable force $f=\frac{dv}{dt}$.

16. Two bodies whose common elasticity is e, moving with given velocities, impinge directly on each other; it is required to determine their velocities after impact.

17. Find the accelerative force when a body P descending down a given inclined plane, draws another Q along a horizontal table by a string acting parallel to the plane of the table.

18. A body descends down a chord of a circle, the plane of which is vertical, and the diameter of which is D, in the time during which, falling vertically, it would describe q; shew that if h be the abscissa of the point from which it sets out, reckoned from the highest point of the circle, the abscissa of the point to which it comes is

$$\frac{h(D+q)^2}{(D-q)^2+4qh}.$$

19. If a body descend down any arc by the action of gravity, the velocity acquired at any point, will be the same as if the body had descended down the same vertical space freely.

20. Obtain in parts of a second the difference of the times of descent down half the cycloid in which a body would oscillate in 1″, and down its chord.

21. Required the time of descending down an inverted cycloid, from the extremity of its base, to a straight line which cuts the cycloid at right angles, and the base at an angle of 60°.

22. Mention several methods in which the problem of the composition of two forces acting on a point has been solved, and shew that they all depend only on physical principles, which have become known by observation or experiment.

TRINITY COLLEGE, June 1832.

1. If two weights acting perpendicularly on a lever, on opposite sides of the fulcrum, have their distances from the fulcrum inversely as the weights, they will balance each other.

2. If any two forces act at the same point, the force which is equivalent to the two is expressed in magnitude by the diagonal of the parallelogram whose sides represent the forces.

3. Two parallel forces act at given points, in a straight line, and in opposite directions, find the magnitude and point of application of the resultant. Explain the result when the forces are equal.

4. In the isosceles wedge find the proportion of the power to the resistance.

5. In the system of pullies, where each pully hangs by a separate string, the power's velocity is to the weight's velocity, as the weight to the power.

6. Find the centre of gravity of any number of bodies considered as points in a plane.

7. Explain what is meant by stable equilibrium, and when a body whose lower surface is spherical rests upon a sphere, find the condition of stability.

8. When a body is supported on a vertical curve, find the conditions of equilibrium.

9. State some of the experiments which shew that at the same place, and near the earth's surface, gravity is a constant accelerating force, and acts equally on all bodies.

10. Explain the terms velocity, moving force, accelerating force, and momentum.

11. When the force is constant the space described in a time $t = \frac{1}{2}ft^2$.

12. Two equally elastic bodies impinge directly upon each other with given velocities, determine their motions after impact, and shew that the motion of their centre of gravity remains unaffected.

13. Find the path of a heavy body projected from a given point, in a given direction, and with a given velocity; and find also the velocity at any point in its course.

14. The time of descent down the arc of an inverted cycloid is independent of the length of the arc.

TRINITY COLLEGE, June 1832.

1. In the bent lever, the power's velocity is to the weight's velocity as the weight to the power.

2. The weights of the voussoirs are as the tangents of the angles which their joints make with the vertical.

3. Find the resultant of any number of forces acting in space, and the equations to the line of its direction.

4. Obtain the equation to the catenary when the chain is acted on by any forces, and thence deduce the equation to the common catenary.

5. Determine the equation of the state bordering upon motion in the inclined plane.

6. State the principle of virtual velocities, and apply it to obtain the conditions of equilibrium of a rigid system acted upon by any number of given forces.

7. Write down and explain the equations which express the conditions necessary and sufficient for equilibrium,

(1). When the system is free,
(2). When the system has a fixed point,
(3). When the system has a fixed axis.

8. When a weight is raised by means of a crank, find the velocity at any point in its circular ascent.

9. When the force is variable, prove strictly that $v = \frac{ds}{dt}$.

10. A quadrant of a circle revolves round the tangent to the middle point of its arc, required the volume of the solid described.

11. Find the centre of gravity of a spherical sector, in which the density of each particle varies as its distance from the centre.

12. Equal forces act at every point in the surface of a given spherical sector, and their directions pass through the centre, find the magnitude and direction of the resultant.

13. If f be the coefficient of friction, what must be the vertical angle of an isosceles triangle, that when placed upon an inclined plane which is gradually elevated, the triangle may begin to slide at the same moment that it also begins to roll over its lowest angular point?

14. Two given beams press against each other upon the ground plane, and rest upon two given curves whose axes are horizontal and coincident, and their vertices at a given distance from each other, find the equations necessary to determine the positions of equilibrium.

15. An uniform chain of given length is placed in a given position on the circumference of a given vertical circle, and is just kept from sliding by a tangential force applied at its highest extremity, compare this force with the pressure sustained by the circumference.

16. Find the number of seconds lost in a day when a seconds pendulum is carried to the top of a mountain two miles high.

17. In a circle two chords are drawn from the extremity of the horizontal radius subtending arcs θ and 2θ. If the time down the chord of $2\theta = n$ times that down the chord of θ, then will

$$\theta = \cos.^{-1} \frac{1}{n^2 - 1}.$$

18. If two bodies be projected from the same point, with equal velocities and in such directions, that they both arrive at the same

point on a plane whose inclination to the horizon is β, and if t, t', be the times of flight and α the angle of projection of the first,

$$\text{then will } t' = t \cdot \frac{\cos.\alpha}{\sin.(\alpha - \beta)}.$$

19. A horizontal heavy radius of a circle moveable about the centre is drawn up into a vertical position by means of a given weight suspended from a line attached to its extremity, and passing over a fixed pully at the extremity of the vertical radius, find the velocity at the end of the motion.

20. In a cycloid if t be the time of descent from the point whose abscissa is the radius of the generating circle to any other point, and if τ be the time down the chord joining the corresponding points in the generating circle, then will

$$2 \tan.^{-1} c\tau = \tan.^{-1}(\sqrt{2} \tan. ct),$$

$$\text{where } c = \sqrt{\frac{g}{l}}.$$

St. JOHN'S COLLEGE, 1816.

1. Explain how the velocity of a body may be expressed numerically; and shew that when bodies have different uniform motions, the spaces described are proportional to the times and velocities jointly.

2. Prove the principle of the composition and resolution of forces.

3. When two equal weights balance each other on a straight lever, the pressure on the fulcrum is equal to their sum. A proof is required.

4. A power P draws up a weight W by a wheel whose breadth is just sufficient to admit one coil of a rope, the thickness of which is $2r$, so that the rope perpetually coils on itself. Neglecting the excess of weight of one rope on the side of the wheel over that on the other, find where W has acquired the greatest velocity.

5. If a weight be sustained on an inclined plane and made to describe a small space, shew that the velocity of the power is to that of the weight : : the weight : the power.

6. From what point must a thin wire in the form of a cycloid be suspended, so that when loaded at each end with half its own weight it shall rest with its base perpendicular to the horizon?

7. Elasticity being perfect, and A, B, C being the weights of three balls in the order of their magnitudes, A strikes B at rest with a given velocity and drives it against C; the distance between B and C being given, and the velocity of A, find where A will overtake B again.

8. Find a point in the circumference of a vertical circle to which a body may fall down an inclined plane from the centre in the same time that another would fall down the diameter.

9. Prove from a property of the sphere, that the time down any chord of a circle whose plane is inclined to the horizon, is equal to the time down the diameter.

10. A thin conical stick being laid in a polished hemisphere, find where it will rest in equilibrium (both ends resting on the surface) and when made to slide, what curve will its centre of gravity describe?

11. In what position of a ladder does a man raising it sustain the least weight?

12. A body falls from a tower 200 feet high. Find the time of falling through a part whose length is two-thirds of its height, and which is so situated that its extremities are equi-distant, respectively from the top and bottom of the tower.

13. Three unequal poles connected at their upper ends and resting their lower on the ground in a triangle, support a weight. Compare the pressures on them in the direction of their length.

14. The time of an oscillation in a cycloid : time down its axis :: circumference of a circle : diameter.

15. A straight lever carries at one extremity a given weight, and to the other is attached a chain, which reaches to the ground, and lies with part of its length loosely coiled up. Find in what position the lever will rest.

16. The distance of the centre of gravity of a pyramidal surface from any one of the planes of which it consists, is one-third of the distance of the opposite vertex from that plane.

17. Explain the method by which the probability of hitting a given mark by a shot fired at random may be determined, the velocity of projection being given; and shew, that when the mark is a horizontal straight line directed to the point of projection, having its farther extremity at a distance equal to the greatest horizontal range, and its length equal to half that range, the odds against hitting it are 2 to 1, the shot being fired in its plane.

18. A cubical solid rests in equilibrium between two inclined planes, containing an acute angle; determine its position, and compare the pressures on the supporting planes. Prove also, that the ratio of these pressures is independent of the figure of the solid.

St. JOHN'S COLLEGE, 1817.

1. Explain, and exemplify the principle "that action and reaction are equal and contrary."

2. The effects of forces when estimated in given directions are not altered by composition or resolution.

3. G is the common centre of gravity of any number of points A, B, C, &c., any how situated. Join GA, GB, GC, &c.; then if forces, represented in quantity and direction by these lines act at once on G, they will keep it at rest.

4. From a given point without a given circle, draw a plane to the circle, so that the time down it shall equal the time down a given plane.

5. Two forces act in given directions on the arms of a bent lever, and keep each other in equilibrio. Find the pressure on the fulcrum.

6. The centre of gravity of a triangular pyramid is the same with that of four equal bodies placed at its corners.

7. From P to any number of fixed points A, B, C, &c., draw PA, PB, &c. so that $PA^2 + PB^2 + PC^2 +$ &c. $=$ a constant quantity. Shew that P will always lie in a spherical surface, whose centre is the centre of gravity of the points A, B, C, &c.

8. A is let fall from a given point, at the same time that B is projected from the same point along a horizontal plane. Find the path of the centre of gravity.

9. *PRQ* is an inflexible metallic lamina, in the form of a circular arc, of a given weight; *P* and *Q* are two weights, likewise given, fixed at its extremities. Determine the position in which it will rest on an horizontal plane *ARB*. Also, given *P*, find *Q*, so that the arc shall rest in a given position.

10. Elasticity is to perfect elasticity :: $m : 1$. *A* strikes *B*, at rest, and drives it round the circumference of a circle. Find where they will meet after $2x$ and $2x + 1$ strokes, the space described, and time elapsed. Shew that after an infinite number of strokes, the velocity of *A* will be the same as if the bodies were perfectly elastic, and of *B*, as if perfectly hard, and one impact only had taken place.

11. In the last problem, suppose elasticity perfect, and find the ratio of $A : B$, so that they shall continue to meet alternately 180° and 360°, from their last point of concourse.

12. Elasticity being perfect, if the number of mean proportionals interposed between two bodies *A* and *X*, be increased without limit; determine the ratio of *A*'s velocity to the velocity thus communicated to *X*.

13. The times of descent down chords of a circle drawn to the extremity of a vertical diameter are equal.

14. *A* moves in the circumference of a circle: *B* is placed without it. Find the path of their centre of gravity.

15. A given weight *P*, draws up (by means of an extremely thin string passing over a fixed pulley) a chain, of an indefinite length, loosely coiled up on an horizontal plane; find *P*'s velocity at any point, and also the point where *P* will begin to re-ascend.

16. Between two parallel planes, perpendicular to the horizon, project a body from a given point so as to return to the hand, after *n* reflections.

17. *DBV* is a semicycloid; from what height *AB* must a body fall on the given point *B*, that it may be reflected into the vertex.

18. On a smooth cycloidal lamina (vertex upwards) is laid a chain, having its upper extremity upon the vertex. Find the time of its running down the whole curve and the velocity acquired, and shew how the same principles may be applied to any rectifiable curve.

19. Elasticity : perfect elasticity :: m : 1. Find the point in a given horizontal line, from which, if a ball be dropped, it shall, after striking a given inclined plane, be reflected to a given point.

20. A and B are two pullies, A fixed, B moveable, and P draws up W; define the circumstances of the motion, when the inertia of A and B is taken into consideration.

21. Make a body oscillate in a given cycloid.

St. JOHN'S COLLEGE, June 1818.

1. If a body be kept at rest by three forces, and lines be drawn equally inclined to the directions in which they act, forming a triangle, the sides of this triangle will represent the quantities of the forces.

2. The same weight is weighed at the two ends of a false balance, and it is observed that the whole gain is $\frac{1}{n}$th part of the true weight; to determine the distance of the fulcrum from the middle point of the lever.

3. If two equal forces sustain each other by means of a string passing over a tack, shew that the pressure upon the tack is to either of the forces as the sine of the angle at which the forces act to the sine of $\frac{1}{2}$ the same angle.

4. The space described by a falling body in the n^{th} second : space passed over in the last second except (n) :: a : b; find the whole space described.

5. If three forces be represented by the three adjacent sides of a rectangular parallelopiped, the compound force is the diagonal; find this force in terms of the other forces, and shew that the sum of the squares of the sines of the angles which each force makes with the compound force is a constant quantity.

6. From the two ends of a vertical straight line two bodies are at the same instant projected towards each other. Find their distance from each other when $\frac{1}{n}$th part of the time in which they would meet, is elapsed.

7. AP, AQ are the directions of two forces P, Q, and AR that of their compound force R, and FE, KG are drawn in any direction parallel to each other, then

$$P : Q : R :: EG.DF : FK.DE : DH.FE.$$

8. The same notation remaining, if any point E be taken in the same plane with P, Q, R, and the perpendiculars EF, EG, EH, be drawn, then $Q.EG \pm P.EF = R.EH$.

9. In a system of (n) moveable pullies, where each pulley hangs by a separate string, and the strings are parallel, if the weights of the pullies, reckoning from the one nearest to W increase in a geometric progression, whose common ratio is (2), when there is an equilibrium $P = \frac{W}{2^n} + \frac{B}{3} . (2^n - 2^{-n})$; B being the weight of the lowest pulley. Find likewise (n) from this equation.

10. A given weight P draws another Q up an inclined plane by means of a thread running parallel to the plane, and the force stretching the string is $\frac{1}{n}$th part of the descending weight; to determine the plane's inclination to the horizon.

11. In the Swedish steelyard the body to be weighed and the constant weight are fixed at its extremities, and the fulcrum is moveable. If then (n) bodies in arithmetic progression are weighed in succession, and the two first are w and w', determine the distance of the fulcrum from either end when the last body is suspended.

12. Compare the time down any arc of a given cycloid with the time down the corresponding chord.

13. If a line be drawn from each extremity of the axis major to any point of an equilateral hyperbola, having its plane and axis vertical, the times of descent down these lines are equal.

14. If the plane and axis of a cissoid be vertical, to determine the line of quickest descent from the curve to the farther extremity of the diameter.

15. Shew from the phenomena of pendulums that a falling body descends through $16\frac{1}{12}$ feet in the first second of time.—Does your proof suppose the body to fall in vacuo?

16. AP, BQ are the directions of two parallel forces P, Q, which sustain each other on the equal arms of the bent lever AFB. Draw FD perpendicular to AB, and FC parallel to AP or BQ, then

$$P + Q : P - Q :: \tan.\frac{AFB}{2} : \tan.\frac{CFD}{2}.$$

17. If θ be the angle which the direction of projection makes with the horizon, (a) and (b) the vertical and horizontal distances of an inclined plane from the point of projection, and (h) the height due to the first velocity, the time of flight (t) will be determined from this equation,

$$t^2 - 2\sqrt{\frac{h}{m}}.\left\{\sin.\theta - \frac{a}{b}.\cos.\theta\right\}.t - \frac{a}{m} = 0.$$

18. A perfectly hard sphere A moves with an uniform velocity (v) along the line IC, another perfectly hard sphere B is so situated that A impinges upon it when its centre arrives at G; calling the $\angle DGC$ (θ),

$$\text{the velocity of } A \text{ after impact} = v\sqrt{\left\{\sin.^2\theta + \left(\frac{A}{A+B}\right)^2.\cos.^2\theta\right\}}$$

19. A body urged by gravity descends in the quadrant of a circle, and is at the same time acted upon by a repulsive force placed in the lowest point varying inversely as the $(\text{dist.})^2$; to find the velocity of the body at any point of its descent, and to determine its positions when at rest, and when its velocity is the greatest.

20. $ACBDG$ a perfectly flexible chain of given length fastened at A passes over the pulley B, which is placed close to A. The excess of DG above BC at the commencement of the motion being given, to ascertain the velocity of the chain after a given portion of it has been drawn over the pulley.

21. Find the centre of gravity of the sector of a sphere.

22. Find the centre of gyration of an ellipse revolving round its centre in its own plane.

St. JOHN'S COLLEGE, May 1823.

1. Define force and its measure, and determine that of gravity from observing the time of an oscillation in a small circular arc.

2. In what position will the weight which a given power supports, by means of a wheel and square axle, be a mean proportional between the greatest and least weights supported on the same machine.

3. If two forces inclined at angles α and β to the arms a and b of a straight lever, not attached to its fulcrum, balance, then

$$a : b :: \tan.(\beta) : \tan.(\alpha).$$

4. In imperfectly elastic bodies the relative velocity before impact : relative velocity after :: compressing force : force of elasticity.

5. If two sides of a billiard-table be inclined at an angle of 5°, and a perfectly elastic body be projected against one of them at an angle of 7°, find after how many reflexions it will cease to approach the angle.

6. If a regular hexagonal canal be placed with two opposite angles in a vertical line; the velocity acquired in falling from the highest to the lowest point : velocity acquired in falling freely down the same height :: 5 : 8.

7. If a cylinder be placed with its axis horizontal, find the greatest distance to which it may be produced, so that a bullet fired from the one end with a given velocity may just pass through it.

8. If two equal parabolas be placed with their axes in the same vertical line ABD, then, if a body fall down the plane BEF and the ordinate FD cut BE in G,

time down BE : time down EF :: $DG : GF$.

9. Prove that when a system is in equilibrio, the centre of gravity is the highest or lowest possible; and hence deduce the position of equilibrium of the two equal beams AC, BD, which, revolving in a vertical plane round the points A and B in the horizontal line AB, support a given weight on the string joining their summits.

10. Find the velocity acquired by the middle point of a rod in falling from a vertical to a horizontal position, the bottom being prevented from sliding.

11. If a body be drawn up a cycloidal canal placed with its axis vertical, by means of an equal weight passing over a pulley at the highest point of the arc; the time of ascending to the highest point : time of $\frac{1}{2}$ an oscillation :: $\sqrt{2}$: 1.

12. Given the weight which a man six feet high can support with his arm horizontal; find the height of a man who can only support his arm in that position.

St. JOHN'S COLLEGE, May 1827.

1. If a rotatory motion be communicated to a body, and it be then left to move freely, shew that the axis of rotation will pass through the centre of gravity.

2. A beam PQ of uniform density and thickness hangs by two strings AP, BQ, from two fixed points A, B. Shew that when there is an equilibrium, the tensions of the strings are inversely as the sines of the angles at P and Q.

3. AB is a vertical line of given length. Find the locus of the point P, so that the square of the time down AP together with the square of the time down PB may be always constant.

4. The time of descent to the lowest point in a small circular arc : time down the chord :: circumference of a circle : four times its diameter.

5. A given conical beam rests with its vertex against a smooth vertical wall, and the base is sustained by a known weight fastened to a string which passes over a fixed pulley. Required the position of the beam when at rest, and the pressure against the wall.

6. A paraboloid rests with its vertex upon that of a given hemisphere. Find the length of its axis so that it may all but fall.

7. Find the centre of gravity of a parabola cut off by an ordinate to any diameter.

8. Two elastic bodies move in opposite directions with equal momenta. Shew that the difference of the products of each body and the velocity of the other before impact : sum after impact :: relative velocity before : relative velocity after.

9. When P raises W by means of the wheel and axle, given P and W and the radius of the wheel; find that of the axle so that the axis may sustain the least possible pressure.

10. Two equal weights are connected by a string of uniform density and thickness which passes over a fixed pulley, and the string when suspended freely will just support a weight (W). The whole being put in motion, find the time elapsed before the string breaks.

11. An elastic ball being projected obliquely upwards is continually reflected by a perfectly hard horizontal plane, and the sum of the areas of all the parabolas described : area of the first :: 8 : 7; find the elasticity of the ball.

12. A hollow sphere, whose external and internal radii are known, rolls down a given inclined plane. Find the inclination of another plane of the same length so that it may slide down it in the same time.

13. An isosceles right-angled triangle ABC is suspended at the right angle A, and its side AB ($4l$), is kept vertical by a ring at B. An angular velocity (ω) being communicated to the triangle round AB, shew that there will be no pressure at B if $\omega^2 = \frac{g}{l}$.

14. If two bodies be projected at equal distances from a plane to which they are attracted by a force varying $\frac{1}{D^3}$, and with velocities which are inversely as the sines of the angles which the directions of projection make with the plane, prove that their common centre of gravity will describe a conic section.

15. Two bodies P and Q connected by an inflexible rod and acted on by gravity move in the circumference of a vertical circle, find the tension of the rod in any position.

St. JOHN'S COLLEGE, Dec. 1829.

1. If two forces, acting on the arms of any lever, keep it at rest, they are inversely proportional to the perpendiculars drawn from the fulcrum on their directions.

2. If a point be kept at rest by three forces, acting upon it at the same time, any three lines, which are in the directions of these forces, and form a triangle, will represent them.

3. Any weights will keep each other in equilibrio on the arms of a straight lever, when the products, which arise from multiplying each weight by its distance from the fulcrum, are equal on each side of the fulcrum.

4. Find the ratio of P to W in a system of n pulleys, where each pulley hangs by a separate string, and each string is attached to the weight.

5. Find the ratio of P to W when they sustain each other upon two inclined planes having a common altitude, by means of a string, which is parallel to the planes.

6. Find the ratio of P to W in the wedge.

7. Prove the third law of motion.

8. In the direct impact of two imperfectly elastic bodies, find the velocity lost by A. And if A impinge upon B at rest, find the ratio of A to B, that A may remain at rest after impact.

9. Define accelerating force. How is it measured? Prove that the space described by a body uniformly accelerated from rest, is equal to half the space it would describe in the same time with the last acquired velocity continued uniform.

10. The length of the arc of a cycloid is double of the corresponding chord of the generating circle.

11. Find the time of an oscillation in a cycloid.

12. Find the equation to the curve described by a body projected in any direction, not vertical, and acted upon by gravity.

13. Find the range on an inclined plane passing through the point of projection: and the greatest height of the projectile above the plane.

St. JOHN'S COLLEGE, Dec. 1830.

1. If two forces, acting on the arms of any lever, keep it at rest, they are inversely proportional to the perpendiculars drawn from the fulcrum on their directions.

2. The effects of forces, when estimated in given directions, are not altered by composition or resolution.

3. Find the proportion between the power and weight in that system of pullies, in which each string is attached to the weight; the strings being parallel.

4. If P and W be two forces in equilibrium on a lever, and the whole be put in motion round the fulcrum, prove that at the beginning of the motion $\frac{P\text{'s velocity}}{W\text{'s velocity}} = \frac{W}{P}$, the velocities of P and W being estimated in the directions in which they act.

5. In a system of bodies, given each body and its perpendicular distance from a fixed plane, find the perpendicular distance of the centre of gravity of the system from the same plane.

6. Prove that the higher the centre of gravity of a body is, *cæteris paribus*, the more easily it is overturned.

7. Write down the three laws of motion, and prove the third law.

8. In the direct impact of two bodies whose elasticity is (e), find their velocities after impact. Explain what is meant by (e).

9. Prove the equations $v = ft$, $s = \frac{1}{2}tv$; (v) being the velocity acquired, and (s) the space described in the time (t), by a body accelerated from rest by an uniform force (f).

10. If a body be projected downwards with a velocity (u) in the direction of an uniform force (f), find its velocity after having described a space (s).

11. Find the time of oscillation in a cycloid.

12. Find the equation between horizontal and vertical co-ordinates to the curve described by a body projected with a given velocity in a direction inclined at a given angle to the horizon.

13. A body being projected at a given angle with the horizon, find the range and time of flight on a given inclined plane passing through the point of projection, and determine the greatest height to which it rises above the plane.

St. JOHN'S COLLEGE, May 1831.

1. If a body descend from rest down any curve by the action of gravity, the velocity acquired at any point is equal to that which would be acquired in falling from rest through the same perpendicular height.

2. State the principle of virtual velocities, and prove it in the case of the single moveable pulley with strings not parallel.

3. If (n) bodies P, $2P$, $3P$, $4P$, &c. be placed at equal distances along a straight lever of given length, find the point upon which they will balance.

4. A beam of given length and thickness rests in a horizontal position with its extremities on two props; and a weight equal to the weight of the beam, hung from its middle point, is just sufficient to break it at that point; what must be the length of the beam between the props, when it will just break in the same point by its own weight?

5. A given uniform beam, having one extremity on a smooth horizontal plane, leans over the top of a given vertical post, and is kept from sliding down by an obstacle placed at its lower extremity. Find the horizontal pressure against this obstacle.

6. If a body descend from rest down a quadrant of a circle by the action of gravity, find at what points of the descent it must be reflected by a horizontal plane, that the range upon this plane may be equal to the perpendicular height fallen through.

7. A globe of given dimensions being projected up a given smooth inclined plane, describes (a) feet upon that plane; how far will it ascend when the plane is rough, the body being projected with the same velocity?

8. The length of an uniform elastic string, when unstretched, is equal to the radius of a given circle; but when laid along the circumference of this circle placed vertically, beginning from the highest point, it extends over an arc of 60°. Find its extensibility.

9. Three perfectly elastic balls A, B, C, are placed at the three angles of a plane triangle, of which the angles are known. Compare the magnitudes of the balls, when A impinging obliquely on B with

a given velocity is reflected so as to strike C, and thence reflected to its first position; the lines drawn from the centres of the balls to the points of impact, being respectively perpendicular to the opposite sides of the triangle.

10. Two bodies, P, and Q, connected by an inflexible rod without weight, rest upon the inner surface of a hollow paraboloid, the axis of which is vertical. Find the position of equilibrium.

11. Find the radius of the globe, which must be attached to the extremity of a second's pendulum, so that the time of vibration may be increased by $\frac{1}{n}$th part of a second.

12. A body, placed at a given altitude above the horizon, is acted upon in a vertical direction by gravity, and in a horizontal direction by a force always proportional to its distance from the horizon. Supposing the body to set out from rest, find its path, and determine its nature and dimensions.

13. Two equally elastic balls descend at the same instant down the arc of a cycloid, from the opposite extremities of its horizontal base. Find the whole space described by them between the 1st and $(2n+1)^{\text{th}}$ impacts.

14. A given uniform rod, acted upon by gravity alone, vibrates in a vertical plane about a horizontal axis passing through one of its extremities. Supposing the rod to be placed horizontally at first, find the quantity and direction of the whole pressure on the axis, in any position of the rod.

St. JOHN'S COLLEGE, May 1832.

1. If four forces be represented in magnitude and direction by lines joining the angles of a triangular pyramid with its centre of gravity, they will keep a particle placed at that point in equilibrium.

2. When a body is projected vertically upwards with a given velocity, determine its velocity after ascending through a given altitude.

3. If a body be projected down a plane inclined at 30° to the horizon with a velocity $= \frac{3}{4}$ of that due to the height of the plane, the time down the plane will equal the time down its vertical height from rest.

4. If R be the resultant of the forces P and Q acting on a point, and r, p, q, be the perpendiculars on their directions drawn from any point in the plane in which they act, then $Rr = Pp + Qq$.

5. The accelerating force on the centre of gravity of two bodies P and Q moving vertically, and connected by a string passing over a fixed pulley $= \left(\frac{P-Q}{P+Q}\right)^2 . g$.

6. Find the centre of gravity of a circular arc; and thence determine the position of a walking stick suspended by the extremity of its handle, which is a semicircle.

7. When an elastic string is suspended vertically from one extremity, the upper half is lengthened three times as much as the lower.

8. Find the equation to the catenary when acted on by gravity.

9. Determine the velocity with which a perfectly hard wheel must move on a horizontal plane, so as just to surmount a given obstacle.

10. A body P acted on by gravity moves down a semicycloid; shew that if PQ be drawn horizontally to meet the circle described on the axis of the cycloid, Q moves uniformly with a velocity due to $\frac{1}{2}$ the radius of the circle.

11. Prove that there are generally two directions, in which a body may be projected with the same velocity so as to pass through a given point; but only one when the velocity of projection is a minimum: determine also the minimum velocity.

12. Define the centre of oscillation of a body, and investigate a general expression for its distance from the point of suspension. Determine the radius of oscillation of a sphere round a horizontal axis at a given distance from its centre.

13. A beam of given weight and length is fixed at its lower extremity to a hinge, and rests with the other on the slant face of a

right-angled prism in such a manner that the motions of rotation and translation of the prism are just prevented. Supposing the hinge and the base of the prism to be in the same horizontal plane, find the position of the prism and the coefficient of friction.

14. When a system of forces acts upon a rigid body, determine the condition that they may have a single resultant.

15. A thin cylinder (W) of given elasticity is drawn along a horizontal plane AW by a string wrapped round it, which passing through an orifice A in the vertical plane BAC, sustains a weight P hanging vertically; determine the whole space described by P before the motion of W ceases.

16. Investigate an equation for determining the tension of a bow string, the curvature of the bow being small.

CAIUS COLLEGE, May 1831.

Second Year.

1. If three parallel forces acting on a straight lever produce equilibrium, each is as the distance between the other two.

2. Prove the parallelogram of forces.

What objections may be urged against your demonstration?

3. Find the ratio of P to W in the screw.

4. Find the centre of gravity of a triangle.

If a b c be the sides of the triangle, and h k l the distances of its angular points from the centre of gravity, then

$$a^2 + b^2 + c^2 = 3(h^2 + k^2 + l^2).$$

5. Determine measures for the requisite qualifications of a good balance.

6. Find the conditions of equilibrium for any forces acting in one plane on a rigid body.

7. Enunciate and prove the principle of virtual velocities.

Does your demonstration prove the principle to the full extent of its application?

8. Prove that the volume generated by the revolution of a plane area about a line in its own plane, is = the area × the path of its centre of gravity:

Find the volume generated by the revolution of the area bounded by an entire cycloid, about a tangent to the curve at its vertex.

9. If a chain be acted on at every point by a force tending to or from a centre, the tension varies inversely as the perpendicular from the centre of force on the tangent.

10. Prove the equation $\frac{d^2s}{dt^2} = f$; and hence deduce, that for an uniform force $2s = ft^2$.

11. If two spheres be moving uniformly in straight lines, having given their velocities and contemporary positions; find, by construction, their positions when nearest each other,

(1.) When the lines are in one plane.

(2.) When they are not in one plane.

12. If a body be projected in a given direction and with a given velocity in vacuo, determine the curve described; and find the maximum distance of the body from a given plane.

13. Find the time of the oscillations of a body made to move in the arc of a cycloid whose axis is vertical; and determine the tension of the string.

CAIUS COLLEGE, JUNE 1832.

1. DEFINE the resultant of two forces; and prove that it acts in the direction of the diagonal of a parallelogram, the sides of which represent the forces in magnitude and direction.

2. Find the conditions of equilibrium of a point upon a curve; the forces acting in the plane of the curve.

3. Prove that two parallel forces, equal and contrary, but not directly opposite, cannot have a single resultant; and that a pair of such forces will make equilibrium with any other similar pair, the plane of which is parallel to the plane of the former, and the tendency to produce motion in a contrary direction.

4. A uniform triangular plate is suspended from a point by strings fastened to the vertices of its angles; prove that the tensions of the strings will be proportional to their lengths.

5. Two forces p and q acting at the extremities of the arms a and b of a straight lever make equilibrium; find the pressure on the fulcrum, p acting on the lever at an angle of 60°.

6. Find the power necessary to sustain a weight upon an inclined plane, the direction of the power being given.

What must the direction be that the power should be a minimum, friction being proportional to the pressure?

7. If $y = \text{chord } x$ be the equation to a curve, the distance of its centre of gravity from the axis will be equal to $\frac{\pi}{4}$.

8. Illustrate the general proof of the principle of virtual velocities by its application to the lever.

9. Define 'inertia' and 'weight'; and prove that the inertia of a body is proportional to its weight.

10. State the second law of motion: and mention some experiments by which it is established.

11. Find the motion of two imperfectly elastic bodies, after their direct impact.

12. A body acted on by gravity is projected in a given direction with a given velocity; find the direction and velocity at any time.

13. Find the time of vibration in a circular arc, and the pressure upon the curve.

14. A clock loses $5''$ a day; what change should be made in the length of the pendulum?

QUEEN'S COLLEGE, 1825.

1. Two forces which are to each other as $2 : \sqrt{3}$, when compounded, produce a force which is equivalent to half the greater: required the angle at which they act.

2. Suppose that a cylindrical lever, 18 feet long, rests in equilibrio upon a fulcrum two feet from one end, having a weight of five pounds at the farther, and another of 100 pounds at the nearer end to the fulcrum; what is the weight of the lever?

3. Two equal weights are suspended by a string passing over three tacks, which form an isosceles triangle, whose base is parallel to the horizon, and vertical angle 120°. It is required to compare the respective pressures on the tacks with each other, and with the weights.

4. Required the centre of gravity of three equal bodies placed at the three angles of an isosceles right-angled triangle.

5. An equiangular prism when placed upon an inclined plane, with its axis parallel to the horizon, is just supported; required the plane's inclination.

6. A body falls down (a) feet of a given inclined plane in (b) seconds, another body at the end of (b) seconds is let fall in the line of descent of the first body at the distance (a) + (c) feet from the vertex of the plane; show how the time may be found, in which the first body will overtake the other; a, b and c being given.

7. It is required to divide a given inclined plane into three such parts, that a body descending down the plane may describe each of them in the same time.

8. Two given weights A and B hang over a fixed pulley; required the length of a pendulum that makes one oscillation whilst A descends one foot.

9. A body, dropped from the top of a tower, falls to the ground in the same time that a pendulum twelve inches long makes six oscillations; required the height of the tower, on the supposition that the body falls through the space of sixteen feet in the first second.

10. If two bodies are projected, at the same time, from two given points with given velocities in given directions, required their distance at the end of t''.

QUEEN'S COLLEGE, May 1831.

1. State the three laws of motion, and give an account of the experiments by which the second is established.

2. There will be equilibrium in that system of pullies in which each pulley hangs by a separate string, if each pulley, the power, and the weight, are all equally heavy.

3. A rod rests with its middle point against a post of half its length and on the ground: show that the horizontal force at the bottom of the rod which prevents the sliding

$$= (\text{weight of the rod}) . \frac{\sqrt{3}}{4}.$$

4. A given thrust is communicated to a smooth plane by means of a rod, which is prevented from sliding by a thread tied to its extremity and making equal angles (θ) with the rod and plane: show that the pressure on the plane $\propto$ tan (θ).

5. State and prove Guldin's Properties, and shew that if the extreme ordinate of a semi-parabola equals its latus rectum, that the solids generated by the area revolving round a tangent at its vertex, the extreme ordinate, and its axis, are respectively as the numbers 1, $\frac{4}{3}$, $\frac{5}{3}$.

6. A hemisphere is placed with its vertex on the base of another one of equal radius: prove that if the density of the upper one : that of the lower :: 3 : 5, that the whole solid will rest on a horizontal plane in all positions.

7. If a system acted on by gravity be in a state of permanent equilibrium, and then slightly disturbed, the small motion of the centre of gravity is entirely horizontal.

8. From the last question show, without any calculation, that an uniform rod will rest in all positions against a point and on the concave arc of the conchoid of Nicomedes.

9. In the impact of two perfectly elastic balls, the sum of each body multiplied by the square of its velocity is the same before and after the impact: show that the same proposition is also true when the motion is continued through any number of elastic balls.

10. At the two extremities of a string which passes over a fixed pulley are attached respectively three and two equal balls, and thus motion ensues for five minutes when suddenly two of the three balls fall off. Show that the remaining single ball will continue to descend for three minutes longer.

11. If a pendulum when carried to the top of a mountain is observed to lose in a given time just twice as much as it does when taken to the bottom of a mine in the neighbourhood—show that the height of the one equals the depth of the other.

12. The moment of inertia of a spheroidal surface of small ellipticity about its axis $= \frac{8}{3}\pi b^4 . \left(1 + \frac{4}{5}\epsilon\right)$ where $\epsilon = \frac{a-b}{b}$.

QUEEN'S COLLEGE, MAY 1832.

1. GIVEN the sum of two forces which act on a point and the angle which each of them makes with the resultant, to find the resultant.

2. If one arm of a balance be slightly longer than the other, the error in the weight will vary as this slight error in the length.

3. If the same weight be again used in the other scale, the loss upon the whole will vary inversely as the square of the length of the arm.

4. A beam rests with one end upon a circle and the other upon a parabola; find the position of equilibrium.

5. $AEBC$ is the quadrant of an ellipse of which AB and AC are the semi-axes: find the position of the centre of gravity of the portion BEC cut off by the quadrantal chord BC: shew that its approximate distance from $AC = \frac{7}{12} BA$.

6. The base of an inclined plane is horizontal and an uniform chain is laid over the plane, reaching either way to the base: prove that the chain has no tendency to move.

7. AC, BC are two uniform rods of equal lengths and perpendicular to each other; but the density of BC : that of AC :: $\sqrt{3} : 1$: prove that they will rest vertically, when the right angle C is placed on a horizontal plane and BC inclined at 60° to the horizon.

8. If any number of forces act upon a rigid body, they are in all cases reducible to a single force and a single couple: prove that the single force passes through the fixed pivots in Roberval's balance.

9. Find the momentum of inertia of an elliptic quadrant about an axis through the extremity of its major axis perpendicular to its plane.

10. An imperfectly elastic ball is let fall vertically upon a parabolic curve placed in a vertical plane, find the point where it must strike the curve so that after reflexion it may pass through the vertex.

11. If a material point, kept at rest by any forces, be slightly disturbed, so that the resulting force of restitution is in the direction of displacement, prove that this force varies as the displacement.

12. Four equal attractive forces are placed in the corners of a square and varying as $\phi(r)$ act on a particle in a diagonal and very near to the centre of the square: shew that the particle will oscillate in the diagonal and the time of one of these small oscillations varies inversely as $\sqrt{\left(\phi'(a) + \frac{\phi(a)}{a}\right)}$ where a is the semi-diagonal.

JESUS COLLEGE, May 1830.

1. Explain what is meant by weight, and show how the weight of any body may be measured.

2. The whole length of the beam of a balance is 3 feet 9 inches. A certain body placed in one scale weighs 9 lbs. and placed in the other weighs 4 lbs: find the true weight of the body, and the length of the arms of the balance.

3. Show how the steel-yard must be graduated.

4. If three forces keep a point at rest, they are to each other as the sines of the angle contained by the other two.

5. A given weight is supported by three props upon a horizontal plane. To find the pressure on each: the lengths of the props and the distances at which they stand being given.

6. In toothed-wheels, the moment of P about the centre of the first wheel, is to the moment of W about the centre of the second wheel, as the perpendiculars from the centres of the wheels upon the line of direction of their mutual actions.

7. Show in the case of the wedge that $P : W :: W$'s velocity in the direction of its action : P's velocity in the direction of its action.

8. Find the centre of gravity of a triangular pyramid.

9. When a body is supported on a curve (the curve being in a vertical plane): to find the conditions of equilibrium.

10. Find the centre of gravity of the sector of a circle.

11. In the direct impact of perfectly elastic bodies, the sum of each body into the square of its velocity is the same before and after impact.

12. When a body is accelerated in a straight line by a uniform force, the velocity is as the time from the beginning of the motion.

13. A heavy chain 100 feet long hangs freely over a fixed pully, 60 feet on one side, and 40 feet on the other, find how long the chain will be in quitting the pully; friction and the inertia of the pully being neglected.

14. A body is to be projected from a given point with a given velocity, so as to strike another given point: to find the direction of projection.

15. If a body descend down any curve by the action of gravity, the velocity acquired at any point will be the same as if the body had descended down the same vertical space falling freely.

16. If a pendulum be slightly altered in length, to find the number of oscillations gained or lost in a day.

JESUS COLLEGE, May 1831.

1. If two forces acting upon a point be represented in magnitude and direction by the two sides of a parallelogram, the force which is equivalent to them both will be represented in magnitude and direction by the diagonal.

2. Compare the power and weight in the equilibrium of a system of pullies in which each string is attached to the weight.

3. When there is an equilibrium on the inclined plane if a small motion be given to the parts of the system

$$P \times P\text{'s velocity} = W \times W\text{'s velocity}.$$

4. Define the centre of gravity, and find it in any system of bodies whatever, considered as points.

5. When an uniform chain is acted upon by a centripetal force, the tension at any point varies inversely as the perpendicular on the tangent from the centre of force: prove this, and find the law of force that a chain may hang in the form of an hyperbola, the centre of force being in the centre of the hyperbola.

6. State the three laws of motion, and prove the third.

7. The velocity and motion of the centre of gravity of two bodies is not altered by impact.

8. Find the time of oscillation in a cycloid; if the body descends from the highest point of the cycloid, what portion of the arc will it describe in $\frac{1}{12}$th part of a complete oscillation.

9. When the force of gravity is slightly altered, find the number of seconds gained or lost in a day by a second's pendulum. Apply your result to the case when a pendulum is carried to a given depth below the earth's surface, the force of gravity varying directly as the distance from its centre.

10. A uniform beam of given length rests between two planes inclined at right angles to each other; find the position of equilibrium, and shew whether it is stable or unstable.

11. Two equal weights connected by a string move freely on a semi-circle in a vertical plane, find the accelerating force in any position, the length of the string being equal to the arc of a quadrant of the circle.

12. In the last problem supposing the two bodies begin to move from a position of equilibrium, then the velocity acquired when one of the bodies arrives at the highest point $= \sqrt{\left(ag \,.\, \tan. \frac{\pi}{8}\right)}$, a being the radius of the circle.

13. A weight P is attached to the extremity of an elastic string, the other extremity being fixed, find the curve on which P will rest in all positions.

JESUS COLLEGE, June 1832.

1. If two forces acting perpendicularly on a straight lever, on the same or different sides of the fulcrum, are inversely as their distances from the fulcrum, they will balance each other.

2. Assuming the principle of virtual velocities, prove that a system acted on by gravity and the reactions of the surfaces on which the

particles move, will have its centre of gravity the highest or lowest possible in the position of equilibrium.

3. Shew how to graduate the common steelyard.

4. Find the resultant of any number of forces acting in the same plane upon a rigid body.

5. If a body acted on by a constant force be projected with the velocity u, and v be the velocity after describing a given space s, then $v^2-u^2=2fs$.

6. In the direct or oblique impact of two perfectly elastic bodies, the sum of each body into the square of its velocity is the same before and after impact.

7. Let P be a point equidistant from two parallel planes whose distance is (a); x and y the co-ordinates of a point Q between the planes measured from P, the axis of x being perpendicular to the planes; if a ball be projected from P in a direction making an angle (θ) with the planes such that $\tan.\theta = \frac{na \pm (-1)^n x}{y}$, it will hit Q after n reflexions; the positive or negative sign being used according as the body is projected towards the plane whose co-ordinate is positive or negative.

8. Find the accelerating force when one weight draws another over a single fixed pully, the inertia of the pully being neglected.

9. P and Q are two weights supporting one another on two planes whose inclinations are α and β, by means of a string passing over their common vertex: the greatest and least values of Q will be $\frac{P\sin.(\alpha+\gamma)}{\sin.(\beta-\gamma)}$ and $\frac{P\sin.(\alpha-\gamma)}{\sin.(\beta+\gamma)}$ respectively, $\tan.\gamma$ being the coefficient of friction.

10. The centres of oscillation and suspension are reciprocal.

11. Given one principal axis, find the other two.

12. A uniform chain hanging over any system of curves or tacks in a vertical plane, will rest when the extremities are in the same horizontal plane; prove this and thence find the tension at any point of the common catenary.

13. A uniform beam whose length $= 2a$ (one end of which rests against a smooth vertical plane) always touches a curve whose equation is $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{a}\right)^{\frac{2}{3}} = 1$, the axis of x being in the vertical plane; determine the motion and shew that if V be the velocity of the centre of gravity of the beam, v of the point of contact of the beam and curve, θ the inclination of the beam to the horizon, $v = V\cos.\theta$; find also the position of equilibrium.

14. A sphere attracted to a given centre of force varying as (Dist.) is projected with a given velocity along a plane passing through that centre; friction being such as to prevent all sliding: prove that the path will be an ellipse, and find the velocity when it is a circle.

SIDNEY SUSSEX COLLEGE, May 1830.

1. Define a lever: and prove that if two equal weights act perpendicularly on a straight lever, they may be kept in equilibrium round any fulcrum by the same force as if they were collected at the middle point between them.

2. An iron bar weighs a ounces per inch: find its length when a given weight w, suspended at one end, keeps it in equilibrio upon a fulcrum at a given distance from the other end.

3. (1). If a body at rest be acted upon at the same instant by any number of forces, which are represented in magnitude and direction by the sides of a polygon taken in order, it will remain at rest.

(2). Is the converse proposition true?

4. Let a given weight W be supported by two props AC, BC upon a smooth horizontal plane AB: find the horizontal pressures at A and B, which will prevent the props from sliding outwards.

5. Explain the construction of a system of pullies, in which the weight of the pullies increases the power of the machine: and, supposing the pullies equal, find the equation of equilibrium.

6. Define the centre of gravity: and find it for any number of points in the same plane.

7. Let a uniform beam be supported on two given inclined planes: find the position in which it will rest.

8. Shew under what condition a system of forces, acting any how in space, can have a single resultant: and find the magnitude and direction of that resultant.

9. Find the differential expression for the centre of gravity of a solid of revolution: and apply it to a parabolic frustum.

10. Find the equations to the common catenary.

11. In the direct impact of imperfectly elastic bodies, find the velocities lost by A and gained by B in the direction of A's motion.

12. Find the direction in which a perfectly elastic ball must move, from a given point, so that it may return to the same point after impinging successively on the four sides of a rectangle: and prove that in its course it describes a parallelogram.

13. Let a body be projected with a given velocity, and accelerated or retarded in the line of its motion by an uniform force: find the time in which it will describe a given space.

14. Let two bodies P and Q be connected by a string passing over a fixed pulley, and let P descend: after P has described a given space let a weight w be removed from P, leaving the remainder $(P - w)$ lighter than Q. Trace the subsequent motion.

15. Prove that a projectile describes a parabola: and find the directrix, focus, and latus rectum.

16. Two bodies are projected from the same point with the same velocity, so as to fall again on the horizontal plane passing through the point of projection, at the same point; and the times of flight are to each other as $m : n$. Find the directions of projection.

17. Explain the construction of the cycloid: and make a body oscillate in it.

18. The length of the second's pendulum, in vacuo, is 39·1386 inches: hence find the actual force of gravity.

19. Let two elastic balls oscillate in the same cycloid, one each side the vertical line drawn through the common point of s pension: explain their motions, supposing the descents to begin

(1). At the same time from different altitudes;

(2). At different times from the same altitude.

20. Define friction; and shew how it may be estimated by periment.

HYDROSTATICS.

TRINITY COLLEGE, 1820.

1. The pressure of a fluid against any surface in a direction perpendicular to it, varies as the area of the surface multiplied into the depth of its centre of gravity below the surface of the fluid.

2. A hollow cone without a bottom stands on a horizontal plane, and water is poured in at the vertex. The weight of the cone being given, how far may it be filled so as not to run out below?

3. What must be the magnitude and point of application of a single force that will support a sluice-gate in the shape of an inverted parabola?

4. Find the specific gravity of a body which is lighter than the fluid in which it is weighed.

5. If the specific gravity of air be called m, that of water being 1, and if W be the weight of any body in air, and W' its weight in water, its weight in vacuo will be

$$W + \frac{m}{1-m}(W - W').$$

6. Three globes of the same diameter and of given specific gravities, are placed in the same straight line. How must they be disposed that they may balance on the same point of the line in vacuo and in water?

7. If a homogeneous hemisphere, floating in a fluid, be slightly inclined from the position of equilibrium; shew that the moment of the fluid to restore it to that position, is not affected by placing any additional weight at its centre.

8. A regular tetrahedron moves with its vertex forwards, in a direction perpendicular to its base: compare the resistance on the oblique planes with that on the base.

9. If the particles of an elastic fluid repel each other with forces varying inversely as the fourth power of their distances, the compressive force on any portion varies as $(\text{density})^2$.

10. Explain the method of measuring altitudes by means of the barometer and thermometer.

11. Two barometers, whose tubes are each l inches long, being imperfectly filled with mercury, are observed to stand at the heights h and h', on one day, and k and k' on another. Find the quantity of air left in each, reducing it to the density when the mercury is at the standard altitude of 30 inches, and supposing the temperature invariable.

12. Construct a common forcing pump; and shew what is the force requisite to force the piston down.

13. In the common sucking pump, given the lowest point to which the piston descends, find the length of the stroke that the pump may work.

14. A cylinder which floats upright in a fluid, is pressed down below the position of equilibrium: when it is left to itself, find the time of its vertical oscillations, neglecting the resistance.

15. A vessel generated by the revolution of a portion of a semi-hyperbola round its conjugate axis, is emptied by an orifice at the centre of the hyperbola: find the time.

16. A close vessel is filled with air n times the density of atmospheric air. A small orifice being made, through which the air rushes into a vacuum, find the time elapsed when the density is diminished one half.

17. A tube of uniform diameter consists of two vertical legs connected by a horizontal branch. When it is made to revolve with a given velocity round one of its vertical legs as an axis, find the height to which the water will rise in the other.

18. Let a spherical body descend in a fluid from rest; having given the diameter of the sphere and its specific gravity relatively to that of the fluid, it is required to assign the time in which the sphere describes any given space.

19. If the density of a medium vary inversely as the distance from a centre, and the centripetal force inversely as any power of the distance from the same centre, a body may describe a logarithmic spiral about that point.

20. If the resistance on a body which oscillates small arcs in a fluid vary as the n^{th} power of the velocity, the difference of the arcs of descent and ascent will vary as the n^{th} power of the whole arc.

TRINITY COLLEGE, 1821.

1. A GIVEN cylindrical vessel full of water, is taken to such a depth beneath the earth's surface that the water sinks $\frac{1}{n}$th of an inch round the edge, find the distance of the surface of the water from the earth's centre.

2. Prove that the centres of pressure and of percussion coincide; and find the centre of pressure of a trapezium, two of whose sides are parallel to the surface of the fluid.

3. If s be the specific gravity of a body, ascertained by weighing it in air and water, and m be the specific gravity of the air at the time when the experiment was made; the correct specific gravity, or that which would have been found if the body had been weighed in a vacuum instead of air, is $s + m(1 - s)$.

4. A square is immersed in a fluid whose specific gravity is to that of the square as 1 to r; shew that when one angle of the square is immersed there will be 12 different positions of equilibrium if r lie between $\frac{8}{32}$ and $\frac{9}{32}$; and when three angles are immersed, that there will be 12 different positions when r lies between $\frac{23}{32}$ and $\frac{24}{32}$.

5. In a floating body, whose transverse section is the same from one end to the other, find the distance of the Metacentre from the centre of gravity of the part immersed; and shew that the equilibrium is stable, or unstable, according as the metacentre lies above or below the centre of gravity of the body.

6. Prove that the velocity with which a fluid issues from a very small orifice in the bottom or side of a vessel is equal to that which a heavy body would acquire by falling from the level of the surface, to the level of the orifice: and shew that this cannot be theoretically true, unless the orifice be indefinitely small.

7. A sphere filled with three fluids whose specific gravities are as 3, 2 and 1, the lighter resting on the heavier, and each of the same altitude, empties itself by a small orifice in the bottom; required the whole time of emptying.

8. If PAQ a circular arc of 120° move in a resisting medium in a direction perpendicular to the chord PQ, shew that the resistance on the arc is equal to the resistance on the triangle PAQ.

9. If the resistance vary as the velocity, and the force of gravity be constant, the times of describing all chords of a circle terminating in the extremity of a vertical diameter, are equal.

10. What angle must a ship's course make with the direction of the wind, and what angle must her sails make with her course, that her motion in a direction opposite to the wind may be the greatest possible?

11. A conical tube of given dimensions partly filled with mercury, has its broad end immersed in a vessel of the same fluid, find the altitude at which the mercury will stand in the tube.

12. If gravity vary as $\frac{1}{(\text{Dist.})^n}$, and the density of the air be proportional to the compressing force, then if a series of distances be taken such that their inverse $(n-1)^{\text{th}}$ powers are in arithmetical progression, the densities at those points will be in geometric.

13. Explain the constructions of an air-pump and condenser.

14. A body oscillating in a cycloid in a medium where the resistance is proportional to the velocity, has at its lowest point a given velocity; find the height to which it will ascend.

15. If the force vary as $\frac{1}{\text{Dist.}}$ and a body describe the reciprocal spiral, find the law of resistance, density, and velocity.

16. A rectangular vessel containing water, is drawn along an horizontal plane by means of a weight which passes over a fixed pulley; find the position of the fluid, and the pressure on any point of the side of the vessel.

TRINITY COLLEGE, 1826.

1. Explain the Hydrostatic paradox, and construct Bramah's Hydrostatic press.

2. A paraboloid filled with fluid stands upon its base, find the pressure upwards against its surface.

3. If a plane be immersed in a fluid, the pressure perpendicular to its surface is equal to the weight of a column of fluid, the base of which is the area of the plane, and the altitude of which is the perpendicular depth of the centre of gravity.

4. A prismatic vessel full of fluid, the section perpendicular to the axis being an equilateral triangle, revolves round its axis with an uniform angular velocity; find the pressure against the sides of the prism, the fluid being supposed to be without weight.

5. Prove the principle of virtual velocities in the case of the equilibrium of any number of forces acting on the surface of an incompressible fluid.

6. Find the diameter of the sphere, which will empty itself in the same time as a cone, through an orifice in its base.

7. Shew that there *may* be six different positions of equilibrium in which an homogeneous triangle may float in a fluid, one angle at least being immersed.

8. Determine the stability of a homogeneous rectangular parallelopiped, floating on a fluid, in a position slightly inclined from one of equilibrium.

9. $ABCD$ is a rectangular parallelogram, of which the side AB is the greatest; it is required to cut off from it such a triangle, by drawing a line from A to a point in CD, that the resistance to the remaining trapezium moving in a fluid in the direction BC may be a minimum.

10. Two barometers of the same given length are imperfectly filled, and the height of the mercury in each on two different days is observed; determine the quantity of air contained in each.

11. Describe the construction of the sucking pump; find the height to which the water will rise after the first stroke, and the conditions necessary for the pump to work.

12. If ρ be the density, and p the pressure, referred to an unit of surface, at any point of a fluid which is solicited by the forces X, Y, Z, in the directions of three rectangular co-ordinates, prove that $dp = \rho(Xdx + Ydy + Zdz)$.

13. From the last equation deduce the following conclusions: (1) that the surface of an incompressible, unconfined fluid in equilibrium, must be perpendicular to the resultant of the forces which act upon it; (2) that each of its level strata must be homogeneous throughout, but that the densities of the different strata may be altogether arbitrary.

14. If the compressing force $\propto (\text{Density})^{1+\frac{1}{m}}$, and gravity $\propto \frac{1}{\text{Dist.}^2}$, determine the pressure at any altitude, and the limit of the height of the atmosphere.

TRINITY COLLEGE, *May* 1831.

1. What is the principle of the transmission of fluid pressure? How is its truth ascertained?

2. The pressure of a fluid downwards against the sides and bottom of any vessel is the weight of the fluid contained in it.

3. The pressure of a fluid has no tendency to communicate lateral motion to a body immersed in it.

4. Define the density and specific gravity of a substance, and give a method of determining the specific gravity of fluids.

5. A piece of wood in vacuo weighs 2 lbs., and when connected with a piece of metal (whose weight in vacuo is 3 lbs., and in water $2\frac{3}{4}$ lbs.), the weight of the compound mass in water is $1\frac{1}{2}$ lb.: determine the specific gravity of the wood and of the metal.

6. Determine the centre of pressure of a given triangle placed with one of its sides perpendicular to the surface of the fluid.

7. A given prismatic vessel is filled with water and covered with a lid moveable about an edge of the top as a hinge. If a given pressure be applied at an orifice in the side of the vessel, required the weight sustained on the middle of the lid when there is an equilibrium.

8. Prove the conditions of equilibrium of a floating body, and apply them to find those of a cone.

9. Assuming the equation of equilibrium of fluids

$$dp = \rho(Xdx + Ydy + Zdz),$$

determine the equation of condition which must be satisfied for the equilibrium to be possible, and shew that it obtains when the fluid is acted on by forces tending to a fixed centre, which are any function of the distance, the density of the fluid being constant, or some function of the pressure.

10. If the pressure of the air vary as the density, and the force of gravity vary inversely as the square of the distance from the earth's centre, find the density at a given height. How does this hypothesis differ from that which really obtains in the earth's atmosphere?

11. Determine the pressure referred to a unit of surface on any point of an homogeneous and incompressible fluid in motion, the motion being steady, that is, always the same for every particle as it passes through the same point in space. Apply the general case to determine the pressure on a plane moving with a given velocity in a fluid in a horizontal direction, gravity being the only impressed force.

12. Determine the motion of a sphere descending in a fluid by the action of gravity, and the circumstances under which the body will, after describing a small space, move on very nearly uniformly.

13. Explain the action of the common pump, and determine the relation between the water raised by the n^{th} and $(n + 1)^{\text{th}}$ strokes of the piston. If the piston do not descend to the fixed sucker at each stroke, shew in what case the water cannot be raised into the barrel of the pump at all.

14. Two thermometers are differently graduated, one of them denotes two particular temperatures by a^0 and b^0, and the other by a'^0 and b'^0, what will the latter indicate when the former indicates n^0?

15. Explain the action of the syphon.

TRINITY COLLEGE, June 1832.

1. State some of the experiments which prove that fluids press equally in all directions.

2. The vertical pressure on the surface of a vessel equals the weight of the superincumbent fluid, and the lateral pressures are equal and opposite.

3. When the particles of a fluid mass are acted upon by given forces, find the equation of equilibrium.

4. Explain what is meant by the centre of pressure; and when the side of a vessel is exposed to the action of a fluid of variable density, determine its co-ordinates.

5. State and explain the conditions that are necessary and sufficient for the equilibrium of a floating body.

6. Explain the principle of the hydrostatic balance; and when a body is weighed in air and water, find its true weight and specific gravity.

7. Shew how to determine the heights of mountains by means of the barometer and thermometer.

8. If a given quantity of air be left in a barometer, determine the true from the apparent height of the mercury.

9. When water issues out of a vessel through a small orifice in the base, explain popularly the form which the issuing fluid assumes a little above and below the orifice.

10. Find the measure of the stability of a floating body.

11. Explain the action of the hydraulic ram.

12. An inverted hemisphere is filled with fluid whose density varies as the depth, find the weight of the fluid, and the position of the elementary zone which sustains the greatest pressure.

13. A current of air proceeding with a given velocity beats against the surface of a hemispherical umbrella, find the force necessary to be exerted in the direction of the axis to preserve the equilibrium, the direction of the current being parallel to the axis.

14. Find the position of a triangular prism floating in a fluid with one of its angles immersed.

15. A closed cylindrical vessel, whose depth is equal to its diameter, is $\frac{1}{3}$ filled with fluid, and placed upon an inclined plane; how high may the plane be raised, before the cylinder will have a tendency to roll over its edge, all sliding being prevented?

16. If the elasticity of the air varied as the $\frac{m+1}{m}$ th power of the density, then would the height of the atmosphere be finite, and equal $(m+1)$ times the height of the homogeneous atmosphere nearly.

St. JOHN'S COLLEGE, 1813.

1. A BODY resting between two fluids has $\frac{1}{m}$ th part in the lower, when it floats on one of them $\frac{1}{n}$ th part is immersed. Compare the specific gravities of the fluids.

2. A circle is just immersed vertically in a fluid, draw that chord from the lowest point on which the pressure shall be the greatest.

3. A tetrahedron moves in a fluid with its vertex foremost; and again with its base foremost. Compare the resistances.

4. The defects of the mercury in the gage of an air-pump, from the standard altitude, decrease in geometrical progression.

5. One side of a vessel of fluid is an isosceles triangle with its axis vertical, and it is retained in its place by fastenings at the angular points. Compare the pressures on them.

6. The fluid issuing from the side of a cylindrical vessel strikes the horizontal plane in the point C. Compare the velocity with which C moves on the horizontal plane with that of the descending surface.

7. In the last question shew how the horizontal plane must be graduated to form a clepsydra.

8. The density of the air at 7 miles altitude, is $\frac{1}{4}$th that at the surface. Having given the height of the mercury in a barometer at the top and bottom of a mountain, to determine its height; gravity being constant.

9. A cylinder of fluid having a small orifice in its base, is attached by a string passing over a fixed pulley, to a weight so heavy that its motion is not sensibly affected by the weight of the cylinder of fluid. Find the time of emptying.

10. A hollow cylinder of given length and closed at the upper end, is filled with air; a weight being attached to the open end, it floats in a fluid at a given depth. Find how far it must be depressed below the surface to be again in equilibrium.

11. The fluid in a vessel whose base is horizontal, is acted on by gravity, and also by a force tending to one of the sides, which varies as the distance from the base. Find the curve the surface will take, and determine whether the internal parts of the fluid can be quiescent.

12. All horizontal sections of a vessel of fluid are semicircles, one side is a vertical plane, down the whole length of which there is a narrow aperture of the form of an inverted triangle. Find the nature of the plane side, so that the velocity of the descending surface may vary as any given power of the depth.

St. JOHN'S COLLEGE, 1814.

1. Give a definition of an elastic and non-elastic fluid, and also of the specific gravity of a body.

2. The pressure of a fluid upon any surface immersed in it is equal to the weight of a cylindrical column of the fluid, whose base is the surface pressed, and altitude the perpendicular depth of the centre of gravity of the surface pressed below the surface of the fluid. Required a proof.

3. A given rectangle is immersed vertically in a fluid, having one side coincident with the surface. It is required to divide it by a line parallel to the surface of the fluid into two parts, the pressures on which may be in a given ratio.

4. A cylinder filled with water is placed upon a wall nine feet in height; at two feet from its base water spouts horizontally through a small orifice, and falls on a horizontal plane at the distance of fourteen feet from the wall. Find the altitude of the cylinder.

5. Having given the quantity of air (p) contained in the air-pump at first, it is required to determine after how many turns a given quantity (q) will be exhausted.

6. The greatest cylinder that can be cut out of a given paraboloid when excavated, is filled with water and placed with its axis vertical. Find the time in which it will empty itself through a small given orifice in its base.

7. The altitude of an homogeneous atmosphere at any point is the same as at the surface of the earth, the temperature being the same. Required a proof.

8. A solid rests in a fluid, the density of which varies as the depth. Shew that at the depth of the centre of gravity of the body, the density of the fluid is the same as that of the solid.

9. A piston weighing (p) pounds, closely fitting a vertical tube whose length is (b), diameter (d), and closed at the bottom, descends by its weight. The barometer standing at (a) inches, and the air in the tube being at first in its natural state, find the distance of the piston from the top of the cylinder when it is at rest.

10. Two vertical cylindric tubes of given diameters and altitudes, one of which is hermetically sealed and the other open at the top, are connected by a third which is horizontal and filled with water, so that the air in the sealed branch may be in its natural state. A column of water of the same base and altitude as the open tube being poured in, determine the space through which it will descend in that branch.

11. Compare the quantities of water discharged in the same time by two equal isosceles triangles cut in the side of a reservoir kept constantly full; the one having its base, the other its vertex upwards, and their summits coinciding with the surface of the fluid.

12. Two cylinders of equal diameters and altitudes open at the top and having their sides vertical are filled with water, and one of them is placed upon the other. A small orifice being made in the base of each, it is required to ascertain the time in which the lower cylinder will be completely emptied.

St. JOHN'S COLLEGE, 1815.

1. Explain what is meant by specific gravity; and shew that if a body be immersed in a fluid of the same specific gravity with itself, it will remain at rest.

2. Prove distinctly that if a prism, whose section is a scalene triangle, be immersed vertically in a fluid, the pressure of the fluid on its sides will not tend to give it any motion of rotation on its axis.

3. Determine the thickness of a right-angled hollow cone of copper, which shall just float with its edge level with the surface of the water, the specific gravity of copper being to that of water $:: 9 : 1$, and the interior and exterior surfaces having a common base.

4. The density of a fluid being supposed proportional to the depth below the surface, the centre of gravity of any solid sustained by it will rest at that depth where the density of the fluid is equal to the mean density of the solid.

5. A cylinder of $\frac{1}{2}$ the specific gravity of a fluid is suspended with its base touching the surface. Being suddenly dropped in, find how deep its upper surface will sink, and how long it will be in sinking.

6. A barrel exhausts a receiver; but, owing to some imperfection in its construction, a barrel full of common air is forced back after every stroke. Find the density of the air in the receiver after any number of strokes, and also after an infinite number.

7. Find the velocity of a body let fall from any height, at any point of its motion (neglecting the resistance); the specific gravity of the body being to that of the air at the earth's surface $:: s : 1$.

8. Given the weight of a bottle when full of air, and when full of water, the weight (in air) of a solid introduced into it, and the weight of the bottle with the solid in it, when filled up with water.

Determine 1st, the specific gravity (s) of the solid, neglecting the weight of the air; and 2ndly, find the correction to be added to s when this is taken into consideration, the specific gravity of the air at the time being n.

9. A logarithmic spiral revolves round an axis perpendicular to its plane, and passing through its pole. Compare the resistance on the curve with the resistance on its greatest radius revolving with the same angular velocity.

10. A cylinder with a small hole in its side revolves round a perpendicular axis, find the nature of the curve which the jet will trace out on the plane of its base, the interior surface of the cylinder being perfectly smooth.

11. State the experiments by which it appears that the air's density is proportional to the compressing force, and then shew that the repulsive force of the particles must be inversely as their distances.

12. Investigate the oscillation of two fluids of unequal specific gravities contained in a bent tube (which would balance each other when their common surface is at the lowest point), the equilibrium being disturbed.

St. JOHN'S COLLEGE, Dec. 1816.

1. Define a fluid. Shew how the specific gravity of a fluid may be found: and find the specific gravity of a solid by weighing it in a fluid specifically heavier than itself.

2. Prove that if a solid be immersed in a fluid whose parts are acted on only by gravity, the weight lost = weight of the fluid displaced.

3. A spherical vessel is filled with water, compare the pressure on its surface with the weight of the fluid.

4. The lower end of a vertical tube, sealed at top, is brought in contact with the surface of a vessel of mercury. Given the receiver and barrel of a condenser under which the apparatus is placed, required the ascent of the mercury in the tube at each successive stroke of the condenser.

5. A cone whose axis is vertical floats between two fluids whose specific gravities are known, and sinks to the depth of half its axis in the heavier. Required its specific gravity.

6. From the top of a hollow glass cylinder closed at both ends projects a small tube which bends downward. The cylinder is made to descend vertically in the sea by a weight attached to the bottom of it. When it reaches the bottom, the weight is disengaged, and the cylinder ascends. The quantity of water in the cylinder being observed, required the depth of the sea.

7. A fluid mass is attracted to two centres of force, the force to each of which varies as the distance. Required its figure when in equilibrio.

8. Explain the construction, and calculate the force of, the hydrostatic press.

9. Required the time of emptying any part of a regular vessel through a small rectangular slit cut in its side, the velocity of efflux being supposed to vary as the $\sqrt{(\text{depth})}$.

10. The side of a vessel of water being loose, required the position, magnitude and direction of a single force that shall keep it at rest, its figure being a semi-parabola with its axis vertical.

11. Construct the common fire-engine, and shew the manner in which it acts.

12. Find the time of libration of water in a cycloidal canal, and shew that it is equal to the time of oscillation in the same cycloid.

St. JOHN'S COLLEGE, 1819.

1. Find the actual value of the specific gravity of a body whose weight and magnitude are given, in the scale in which that of water is unity.

2. If a solid be immersed in a fluid, the whole weight : weight lost :: specific gravity of the solid : specific gravity of the fluid.

3. A spherical vessel is filled with water. Find the horizontal section of the fluid on which the pressure is the greatest. Find also the actual value of the pressure, the whole weight of the fluid being given.

4. A parabola PAQ moves through a fluid in the direction of its axis RA: PR, RQ are normals. Prove that the resistance on the curve : resistance on base PNQ :: sector $RSNT$: triangle RPQ.

5. Shew how the diameter of a capillary tube may be found with great accuracy.

6. A cylindrical bucket floats on a fluid of the same specific gravity as the materials of which it is formed: having given its length, the radii of the internal and external circumferences, find the time of filling through a small given orifice at its base.

7. A prismatic vessel stands on a horizontal plane; it is required to make a small orifice at its side, so that the area of the parabola described by the issuing fluid may be the greatest possible.

8. Having given the altitude of the mercury in the gauge of an air-pump and the capacities of the receiver and barrel: find the number of turns.

9. A conical vessel is filled with a fluid whose density varies as the depth; the pressure on the base being equal to that on the sides: find the vertical angle.

10. In the last question, if the vessel be inverted and an aperture made in the base, find what portion of the fluid will escape.

11. An elastic fluid whose specific gravity at the earth's surface is $\frac{1}{n}$th that of the air is put into a freely expansive balloon without weight; find the height to which it will ascend, supposing the compressing force on the air to vary as the square of the density, that of the fluid as the 4th power, and gravity inversely as the square of the distance from the earth's centre.

ST. JOHN'S COLLEGE, DEC. 1821.

1. DEFINE specific gravity, and from the definition shew, that the weight of a body varies as its magnitude and specific gravity jointly; then compare the specific gravities of gold and silver, supposing a cubic inch of gold to weigh 11 ounces, and a cubic foot of silver to weigh 960 pounds.

2. Explain the construction of a wheel barometer; and find the height of the mercury in a common barometer, supposing the index of the vernier to be between 29·5 and 29·6 on the scale, and the division 7 on the vernier to coincide with a division on the scale.

3. A parallelogram and a triangle of the same base and altitude, move in a fluid with velocities which are as 2 : 3. They are inclined to the directions of their motions at angles of 45° and 30° respectively. Compare the resistances on the planes in directions perpendicular to them.

4. In a cylinder which is three fourths filled with water, an hydrometer is observed to rest at a certain depth. Supposing the vessel filled up with fluid of three times the specific gravity of water, with what weight must the instrument be loaded to make it sink to the same depth in the mixture?

5. A cylinder has some fluid in it. Suppose from a change of temperature the bulk of the fluid to be increased an n^{th} part; what alteration will take place in the pressure on the sides and base?

6. If a be the altitude of a column of water which the air would support, m the greatest distance between the sucker and the surface of the water, the common pump cannot work unless the length of the stroke be greater than $\frac{m^2}{4a}$.

7. In a clepsydra, where the surface of the fluid descends uniformly, the horizontal sections are similar parabolas. Find the nature of the curve which is the locus of their vertices.

8. A cone A and cylinder B of the same specific gravity, base, and altitude, balance each other at the extremities of a straight lever when immersed in a fluid of given specific gravity. Supposing a cone equal to A, cut out from B, and its place supplied by another of half its specific gravity, find what part must be cut off from A to restore the equilibrium.

9. A paraboloid resting on its base is kept constantly filled with fluid; find at what point a very small orifice must be made that the latus rectum of the parabola described by the issuing fluid may be half the latus rectum of the vessel.

10. Let a sphere descend in a fluid from rest; find its velocity at any point of its descent, having given the diameter of the sphere, and the ratio of its specific gravity to that of the fluid; and shew from the result, that very small bodies descending in a fluid acquire (as to sense) their greatest velocities, after descending through a small space.

11. The barrels of an air pump communicate with the receiver of a condenser which is of the same magnitude as that of the pump. The density of the air in the condenser is 3 times that in the pump, which is in its natural state, and a barometer tube having the basin of mercury in the condenser, has its upper end, which is open, in the pump. A piston of the same diameter as the tube, and whose weight = the weight of a column of mercury of the standard altitude, is placed in the tube and suffered to descend; find its place when at rest at first, and after 2 turns, having given the standard altitude (a), the length of the tube ($4a$) and the ratio of either receiver to a barrel 3 : 1.

12. *ABC* is the side of a tetrahedron which is filled with fluid, *AD* is a perpendicular from the vertex *A* on *BC*; the part *ABD* is loose; find the magnitude, point of application, and inclination to the horizon of a single force which will keep it at rest: 1st, when the vessel rests on its base; 2nd, when it rests on its vertex; the base in each case being horizontal.

St. JOHN'S COLLEGE, Dec. 1822.

1. Prove that the perpendicular altitudes of fluids sustained in different tubes by the atmospheric pressure, are inversely proportional to their specific gravities.

2. Compare the specific gravities of two bodies, one of which weighs (10) lbs. in vacuo, and the other (5) lbs. in water; the bulks of the two bodies being respectively 48 and 72 cubic inches, and the weight of a cubic foot of water 1000 ounces.

3. A rectangular parallelogram being immersed vertically in a fluid with one side coincident with the upper surface; it is required to divide it by a line, drawn from a given point in this side, into two such parts that the pressures on them shall be equal.

4. One fourth part of a cubical solid of given dimensions, which floats on the surface of a fluid whose specific gravity is known, is removed by a section parallel to the upper surface, when it is found to rest with the part extant equal to twice the part before immersed. Find the weight of the cube.

5. Some air being left in the tube of a barometer, the mercury stands at the distance (a_1) below the standard altitude; but the basin being placed in the receiver of a condenser, it is found, after (n) turns, to rise to the distance (a_2) above the standard altitude. Supposing the capacity of the receiver to be (p) times as great as that of the barrel of the condenser, shew how the two altitudes of the mercury may be determined.

6. Compare the perpendicular resistances on the sides of a wedge moving uniformly through a fluid in a direction perpendicular to the back, the sides being inclined to the back at 45° and 60° respectively.

7. Having given the law according to which the particles of an elastic fluid repel one another, determine to what root or power of the density, the compressing force is proportional.

8. Prove that the time in which a paraboloid, placed with its axis vertical, will empty itself through a small orifice in it's base, is to that in which the greatest cylinder which can be cut out of the paraboloid will empty itself through the same orifice $:: 4\sqrt{2} : 3$.

9. Shew how to find the quantity of fluid discharged in any assigned time, through a vertical orifice of any given form and dimensions, supposing the vessel to be always kept filled up to the same point: and apply your method to the case where the orifice is triangular.

10. A spherical buoy of given weight and dimensions floats on the surface of stagnant water, with $\frac{1}{n}$th part of its vertical radius below the level of the surface; but when attached to the bottom of a running stream, whose depth is known, by a string of given length, it is observed to be just half immersed. Neglecting the weight of the string, it is required to determine from hence the velocity of the stream.

St. JOHN'S COLLEGE, Dec. 1823.

1. Prove that particles at the same perpendicular depth are equally pressed.

2. Explain the phenomenon of intermitting springs on hydrostatical principles.

3. If a spherical vessel is emptied by a hole at the lowest point, find two horizontal sections, equidistant from the centre, where the velocities of the descending surface are in a given ratio.

4. Given the weights of a body in air, corresponding to the heights of the barometer h and h'; find the weight corresponding to a height h''.

5. Air, whatever be its density, will rush through a small orifice into a vacuum with the same velocity.

6. If an isosceles triangle be immersed in a fluid with its base horizontal, and vertex coinciding with the surface of the fluid, how far must the one side be produced so that joining its extremity with that of the other side the pressure on the whole triangle thus formed may be double that on the isosceles triangle?

7. If two holes be made in the side of a prismatical vessel of water at one half and one third the depth, find where the effluent streams will intersect.

8. A cylinder of known density and magnitude, floats with its axis vertical in a vessel of water placed under a condenser; after how many depressions of the piston will it be elevated by a given quantity?

9. At what angle must a plane be inclined to a fluid, so that the excess of the resistance in the direction of the stream above that which is perpendicular to it, may be a maximum?

10. A portion of the receiver of an air pump is a plane valve opening inwards and kept in its place by a spring acting with a pressure p, less than that of the atmosphere; after how many turns will the external air open the valve?

11. How much must the breadth of a rectangle, revolving in a fluid round its longer side, be increased, so that the resistance to its motion may be the same as when it revolved round its shorter side?

12. A parallelopiped with its sides vertical has one side loose which revolves round a hinge at the bottom, and is kept in its position by a given pressure at a given point; how high may the vessel be filled with fluid before the side will be forced open?

St. JOHN'S COLLEGE, May 1825.

1. If a solid be immersed in one or more fluids acted on by gravity, investigate the conditions of equilibrium.

2. Construct Bramah's press; and determine the requisite thickness and strength of the cylinders to resist a pressure of (p) pounds per square inch.

3. A sphere filled with water is divided by a vertical plane into two hemispheres. Required the position and magnitude of the lateral forces which shall just prevent their separation.

4. Explain the construction and use of Nicholson's hydrometer; and investigate a formula by which it may serve as a barometer.

5. Required the density of the densest fluid in which a paraboloid of given weight and magnitude can float permanently with its axis vertical.

6. A mass is observed to detach itself from a floating spherical iceberg, which begins in consequence to oscillate slowly. The time of these oscillations being noted and the weight of the mass ascertained, determine the weight of the berg.

7. Prove by theory that the velocity at the orifice equal that due to half its depth. Why must the orifice be small? Does this proposition apply to the case of elastic fluids?

8. If t'' = time of emptying an upright cylindrical vessel, the quantity discharged in t'' when the vessel is kept constantly full equals twice the content of the vessel.

9. A cylinder of fluid is placed vertically on a table whose altitude equals that of the cylinder. An orifice is made in the lowest point through which the fluid spouts horizontally, and is received

into (n) equal vessels placed on the horizontal plane. Required the ratio of the quantities discharged into each vessel.

10. A paraboloid of given dimensions filled with air has water forced into it till it is half full. Required the time of emptying through an orifice in the vertex.

11. Compare the resistance on the arc of a cycloid moving in the direction of its axis with the resistance on the same arc moving in the direction of its base.

12. Explain the construction of the flying bridge, and compare the velocity of the stream with that of the boat when set at the best angle for crossing the river.

13. If a cube of elastic fluid be compressed into another cube whose side : side of the former :: n : 1; compare the whole pressure on a side of these cubes.

14. Shew that the quantities of air which issue from a vessel of condensed air into a vacuum in equal times decrease in geometric proportion.

15. Explain what is meant by an homogeneous atmosphere; find its height, and prove it to be the same at all altitudes, gravity being supposed constant.

16. Explain the principle and construction of the steam-engine, and mention the improvements introduced by Watt.

17. A cup when nearly full of water stands vertically; but when a little inclined the whole runs out. Shew how this may be effected.

18. Determine the conditions of equilibrium in the screw of Archimedes.

19. A body acted on by gravity describes a semi-circle. Investigate the law of resistance and velocity. (*Newton*, Prop. 10. Vol. II.)

20. Find the times of vibration of a musical string.

St. JOHN'S COLLEGE, Dec. 1825.

1. The perpendicular altitudes of fluids communicating through a bent tube vary inversely as their specific gravities.

2. Two masses of given specific gravities balance when suspended from the equal arms of a lever in a known fluid; what is the specific gravity of the fluid in which they balance when one of the masses is doubled?

3. The velocity of the water issuing from an orifice, is three times greater than when the pressure of the air is removed from the upper surface of the fluid; what is the depth of the orifice?

4. Explain the use of the spirit level, and find the correction in levelling due to the spherical shape of the earth.

5. A right-angled triangle moves in a fluid in a direction perpendicular to its hypothenuse, and the resistances on the sides in the direction of the motion are as 9 : 1; find the angles of the triangle.

6. A vessel full of fluid in the form of a rectangular parallelopiped stands upright on a horizontal plane; where must an orifice of given dimensions be made, so that the vessel may be overturned? and what is the least height of the vessel for which the problem is possible?

7. Investigate the figure in which the centre of pressure is situated at the depth of two-thirds of the axis, when the vertex coincides with the surface of the fluid.

8. Compare the quantities of air exhausted in $2t$ turns by an air pump with a barrel B with that exhausted in t turns by a pump with a barrel $2B$, the receivers in each case being equal; and shew that short barrels exhaust more rapidly than long ones of the same diameter when the pistons move at the same rate.

9. A barometer suspended by its upper extremity from the arm of a balance will support a weight = the weight of the tube + the weight of the mercury sustained in it, nearly.

10. The jet of a fountain terminates in a right cone pierced with holes, the axis of which is vertical, and so small compared with the height of the reservoir, that the fluid may be supposed to issue with the same velocity; shew that the surface of the issuing fluid will be bounded by a similar cone, and find its vertex.

11. A cylinder closed at both ends is divided into two unequal compartments by rectangular planes which are terminated by the axis and the surface of the cylinder; if one of these planes is fixed and the other which is of given weight admits of being turned round the

axis of the cylinder, find the greatest velocity it can acquire when the two compartments are filled with air of given densities.

12. Explain what the Metacentre is, and find it in a cylinder which floats with its axis vertical.

St. JOHN'S COLLEGE, Dec. 1826.

1. Define specific gravity, and shew how the specific gravity of cork may be determined.

2. A given parabolic arc moves in a fluid in a direction perpendicular to its axis. Compare the resistance on the arc with that on its chord.

3. An oblate spheroid filled with fluid, the density of which varies as the depth, is placed with its major axis vertical. Find that horizontal section which sustains the greatest pressure, and determine also the actual pressure on that section.

4. An n^{th} part of a given cylindrical tube being filled with common air, find how much mercury must be poured in at the top so as just to fill the tube.

5. Given the degree of temperature in one thermometer, find generally the corresponding degree in another which is differently graduated.

6. Compare the resistance on a semi-parabola revolving in a fluid round a tangent at its vertex with the resistance when it revolves round its axis.

7. A spherical vessel filled with fluid revolves round a diameter with a given angular velocity, and the pressure on the internal surface : whole weight of the fluid :: n : 1; required the content of the vessel.

8. One side of a vessel filled with fluid is a trapezium, with two of its sides horizontal, and the remaining two equal. Supposing this kept in its place by fastenings at the angular points, compare the pressures at those points.

9. A paraboloid floats on a fluid of nine times its specific gravity, with its axis vertical and vertex downwards. Shew that when the

equilibrium is that of indifference, the length of the axis : latus rectum :: $3^2 : 2^3$.

10. Two equal semicycloidal tubes are placed with their axes in the same horizontal line, and their bases coincident. One of these being filled with fluid which oscillates in the tubes, it is required to determine its velocity in any position, and also its maximum value.

11. Compare the effect produced by one stroke of a piston, when the steam is admitted into the cylinder during the whole descent, with that which would be produced if it were admitted for an n^{th} part of the descent.

12. A cut is made in the side of a vessel filled with fluid, the density of which varies as the depth, in the shape of a triangle with its base coincident with the surface. Supposing this loose, and to revolve round its base which is known, find its altitude, so that a given force applied at its vertex may keep it at rest.

St. JOHN'S COLLEGE, Dec. 1829.

1. The resultant of the pressures of a fluid on a solid immersed in it, is equal to the weight of the fluid displaced, and acts upwards in a vertical passing through the centre of gravity of the fluid displaced.

2. A ship on sailing into a river sinks 2 inches, and after discharging 12,000lbs. of her cargo rises one inch; find the weight of the ship and cargo. $(SG \text{ sea water}) \div (SG \text{ fresh water}) = 1{\cdot}026$.

3. Find the positions of equilibrium of two equal slender rods forming a right angle floating in a vertical plane, with the angle immersed.

4. Find the metacentre of a solid cone: and also of a hollow cone partly filled with the fluid in which it floats.

5. A hollow vertical prism just filled with heavy incompressible fluid revolves round one edge with a given angular velocity: find the centre of pressure on the base, which is an isosceles right-angled triangle with one of its equal angles in the axis of revolution.

6. A weathercock is formed of two unequal vanes making a given angle with each other, and moveable about their common intersection: find the angle between either vane and the direction of the wind.

7. Find the depth of a mine by means of a barometer, the temperature being uniform throughout.

8. Find the tension of the rope, by which a diving bell is suspended, at any depth below the surface.

9. A barometer tube filled with mercury, and closed by the finger at the open end, is inverted in a cup of mercury: determine the motion of the upper end of the mercurial column, after the finger is removed.

10. Find the least angular velocity of the moveable board of a bellows, that will enable the valve to open.

11. Find the terminal velocity of a ball of ivory descending in water.

Diameter of the ball 6 inches, $(SG \text{ ivory}) \div (SG \text{ water}) = 1{\cdot}825$.

12. Determine the form of the film of water elevated by capillary attraction between the surfaces of two convex lenses touching each other, and having the centres of their contiguous surfaces in the undisturbed surface of the water.

13. A thermometer open at the top, contains 1,200 grains of mercury at 32°, on being exposed to a higher temperature 3·12 grains of mercury are expelled: find the temperature. The expansion of mercury in volume from 32° to 212° being 0·018, and the linear expansion of glass between the same points 0·0008.

14. In De Lisle's thermometer the boiling point is marked 0°, and the freezing point 150°:

What degree of Fahrenheit's corresponds to 138° of De Lisle's?

St. JOHN'S COLLEGE, May 1831.

1. From the principle that when a mass of fluid is in equilibrium, its state of rest is not altered by supposing any portion of it to become solid, deduce the equality of the pressure of fluid in all directions.

2. Find the pressure at any point in a mass of fluid, at rest, and acted upon by gravity. How is the pressure estimated, when it is said to be equal to $g\rho z$, g being the force of gravity, z the depth below the surface, and ρ the density?

3. The pressure on any surface immersed in a fluid is equal to the weight of a column of fluid whose base is equal to the surface pressed, and altitude the perpendicular depth of its centre of gravity.

4. Describe the common thermometer, and shew how it may be filled, and graduated. Also shew how the scales of two differently graduated thermometers may be compared.

5. Explain the construction and use of the Hydrostatic press; and find the pressure produced by a given force.

6. In the common pump, find the tension of the rod corresponding to a given position of the ascending column; also find the height through which the water rises each time the piston ascends; and the least range of the piston that will enable the pump to produce its full effect.

7. Define specific gravity; shew how to compare the specific gravities of air and water; also compare the specific gravities of any solid and a fluid, by weighing the solid in air, and in the fluid; taking into account the weight of the air displaced by the solid.

8. Describe Smeaton's air pump; point out the advantages of its construction; find the density of the air in the receiver after any number of ascents of the piston. Explain the Syphon guage.

9. Find the difference of altitudes of two stations by means of the barometer; obtain a formula for the case where the difference is so small, that the variation of gravity may be neglected.

10. Find the velocity with which an incompressible fluid, acted on by gravity, issues through an indefinitely small orifice in the vessel containing it; also find the time of emptying the vessel.

11. Explain what is meant by the resistance of a fluid on a solid moving in it. When a stream impinges obliquely on a plane, find the force with which the stream impels the plane, estimated in the direction of the stream, and in a direction perpendicular to it. Find the resistance on a solid of revolution moving in the direction of its axis.

12. Find the pressure at any point in a mass of fluid at rest acted on by any forces; and the relation that must exist among the forces in order that the equilibrium may be possible.

13. Find the velocity with which an incompressible fluid, acted on by gravity, issues through a finite orifice in the horizontal base of the vessel in which it is contained.

St. JOHN'S COLLEGE, Dec. 1831.

1. Find that part of the pressure of a fluid on any surface, which acts in a direction perpendicular to a given vertical plane.

2. Find the weight of a stone which when tied to a block of wood, will just cause it to sink in water, the weight of the wood being 523 ounces, $S.G$ wood : $S.G$ water $= 0{\cdot}596$, $S.G$ stone : $S.G$ water $= 2{\cdot}6$.

3. A piece of glass appears to weigh 732·6 grains, when the temperature of the air is 0°, and its pressure equal to that of a column of mercury 30 inches high. How much will it appear to weigh, at the same temperature, when the pressure of the air is equal to that of a column of mercury 28 inches high? The weights are made of platina. $S.G$ platina : $S.G$ water $= 21$. $S.G$ glass : $S.G$ water $= 2{\cdot}4$. $S.G$ air : $S.G$ water $= 0{\cdot}0013$, (thermometer 0°, barometer 29·92 inches).

4. Find the centre of pressure of an equilateral triangle having one side vertical, and one angle in the surface of the fluid.

5. Find the difference between the vertical pressures on the inner and outer surfaces of a diving bell.

6. Find the angle between the gates of a lock, when the pressure of their edges against each other is a minimum.

7. The base of a homogeneous pyramid is an equilateral triangle, its faces are isosceles right-angled triangles.

(1). Find the positions in which it can float with its vertex immersed.

(2). Find the inclination of its base to the surface of the fluid, when a very small weight is placed upon one of the angles of its base, the base of the pyramid being originally horizontal.

(3). Find the density of the heaviest fluid in which it can float permanently with its base horizontal.

8. A hollow sphere is immersed in fluid till the plane of the surface of the fluid passes through the centre of the sphere, find the place of an orifice such that the altitude of the highest part of the jet issuing through it may be a minimum.

9. Fluid is forced into a vertical hollow cylinder till the pressure on its base is equal to twice the pressure on its upper end; the cylinder is then made to revolve round a vertical axis in its surface; find the pressure on the concave surface of the cylinder.

10. A copper wire the diameter of which is $\frac{1}{10}$ inch, will just support 191 pounds without breaking; find the greatest pressure (estimated by the pressure on a square inch) that can be applied to a fluid contained in a hollow prolate spheroid of copper, the semi-axes of which are 4 and 6 inches, and thickness $\frac{1}{100}$ inch, without bursting the spheroid.

11. When two people *A* and *B* descend to the bottom of a lake, in a cylindrical diving bell, the water within the bell is observed to stand one inch lower than when *A* alone descends, the pressure of the atmosphere is equal to the pressure of a column of water 34 feet high, the diameter of the bell is 4 feet, and the surface of the water within it, at the bottom of the lake, is 20 feet below the surface of the lake; find the volume of *B*.

12. Having given the area and velocity of the paddles of a steam boat, and the tension of a rope by which it is towed at a given rate, find the velocity of the boat when propelled by its paddles.

St. JOHN'S COLLEGE, May 1832.

1. Define the weight, mass, volume, specific gravity and density of a body, and find the relations existing between them.

2. The common surface of two fluids that do not mix is a horizontal plane.

3. Find the vertical pressure of a fluid on any surface.

4. The pressure of the air at a given temperature varies inversely as the space it occupies.

5. Find the difference of the altitudes of two stations by means of the barometer.

6. Find the velocity with which an incompressible fluid acted on by gravity issues through an indefinitely small orifice in the vessel containing it.

7. Find the force with which a stream impels a plane, 1st. when the plane is perpendicular to the direction of the stream, and 2ndly., when situated obliquely to it.

8. Explain the method of filling and graduating a thermometer; and find the number of degrees of the centigrade, corresponding to 60° of Fahrenheit's thermometer.

9. Describe Nicholson's hydrometer; and by means of it, compare the specific gravities of a solid and fluid, or of two fluids.

10. Find the time of emptying a vessel through a very small orifice.

11. Find the metacentre of a cone floating with its axis vertical.

12. Describe Watt's steam engine.

13. Determine the pressure at any point in a mass of fluid at rest acted on by any forces.

CAIUS COLLEGE, May 1830.

1. Define a fluid, and explain what is meant by the equal distribution of pressure.

2. The altitudes of two heavy homogeneous fluids in communicating vessels, measured from their common surface, are inversely as their specific gravities.

3. A plane area is immersed in a heavy fluid in a position inclined to the horizon, find the co-ordinates of the centre of pressure, and shew that that point is lower than the centre of gravity.

4. Find the positions of equilibrium of a triangular prism floating in a heavy fluid.

5. Investigate the equation for the equilibrium of a fluid, each of the molecules of which are acted on by given accelerating forces: (what is to be inferred if $Xdx + Ydy + Zdz$ be not a complete differential?)

6. Determine the whole pressure on a paraboloid containing a heavy fluid revolving uniformly round its axis.

7. Find the time of emptying a vessel through a very small orifice, (1) into vacuo, (2) into another vessel communicating with the former by this orifice and having its base on a level with the orifice.

8. Describe the hydrostatic balance and the diving bell; find the height of the water in the bell at a given depth below the surface of the fluid.

9. Find the length of the hydrophorous arc in the screw of Archimedes.

10. Determine the resistance on the surface of a sphere moving through a homogeneous fluid.

11. Find in general the metacentre and stability of a floating body.

12. Investigate a formula for measuring heights by the barometer and thermometer.

13. Determine the motion of a heavy fluid through any orifice, supposing the sections of the fluid to preserve their parallelism while descending.

14. Find the time of the small oscillations of a heavy fluid in a narrow circular tube.

15. Investigate the equations which express the continuity of an incompressible fluid mass in motion, and the permanence of the density of its molecules.

CAIUS COLLEGE, May 1831.

1. Shew how the property of equal distribution of pressure is proved to belong to fluids.

2. Any horizontal section being made of a vessel containing fluid of variable density; the density throughout that section is uniform.

3. If a mixture be made of two substances, first in equal quantities, and next in equal weights, the specific gravities of the mixtures may be found, and they will be, in the first case an arithmetic mean, in the second, a harmonic mean between the specific gravities of the simple substances.

4. A semicircular tube is filled with equal quantities of fluid of given specific gravities; what quantity of the lighter fluid will be expelled before the equilibrium is established?

5. A vessel in the shape of a truncated paraboloid is filled with water and placed on its two ends successively, supposing the base in each case to be moveable; compare the pressures necessary to be applied in each case to sustain the base.

6. Determine the general expressions for the co-ordinates of the centre of pressure; and apply them to determine that point in a semi-ellipse, having its vertex in the surface of the fluid.

7. Determine the conditions of equilibrium of a floating body.

8. A cone of given dimensions and weight is immersed in water, a certain small weight is added; find the depth the cone sinks, and the time of its oscillation when that weight is removed.

9. Describe the manner of ascertaining the specific gravity of a solid, by Nicholson's hydrometer.

10. Find the equation to the surface which a fluid assumes when acted upon by any forces; and thence determine the nature of the surface of the water in a cylindrical vessel revolving with a given angular velocity.

11. The density of the air decreases in geometric proportion, through equal small increments of altitude. Show this, and determine the correction for the variation of temperature.

12. A hollow cone is filled with equal quantities of two given fluids which do not mix; compare the times in which each fluid passes through an orifice in the vertex.

13. A plane is inclined to the direction of a stream at an angle ϕ; find the resistance to its motion in a direction perpendicular to that of the stream.

14. Find the density of the air, in the receiver of an air pump, after a given number of turns, by observation on the gage.

15. A certain degree of temperature in one thermometer is to be reduced to an equivalent degree in another of different graduation? Investigate a method for effecting this.

16. Explain the principle of the barometer, and investigate an equation for determining the heights of the mercury, when the barometer is not entirely free from air; the law of repulsion in the particles of air being $\frac{1}{d^n}$.

17. In the common pump, find the height to which the water will rise after a given number of descents of the piston.

CAIUS COLLEGE, June 1832.

1. Find the general equation of equilibrium of any fluid.

In heterogeneous fluids shew that the density is uniform throughout all surfaces of equal pressure.

Prove that the resultant of the forces at any point in the external surface of a fluid at rest is in the direction of a normal at that point.

2. If a vessel contain any number of fluids which do not mix, the common surface of every two will be horizontal.

3. Find the pressure on a plane immersed in a fluid which is acted on only by gravity.

Find the pressure on a semi-parabola immersed vertically with the angular point coincident with the surface, and the axis making an angle of 45° with it.

4. Investigate the conditions of equilibrium when a body floats on a fluid.

If the body be pressed downwards through a very small space and then let go, determine the motion.

5. If the elastic force of air be proportional to its density, prove that the atmosphere must be of infinite extent.

How is the density affected by the temperature?

6. Find the time in which a vessel will be emptied through a given small orifice.

A cone is filled with equal weights of two given fluids which do not mix; find the time in which it will be emptied through a small orifice in the vertex.

7. Explain the action of the steam engine.

8. Account for the phenomena of the trade winds.

9. A uniform rod vibrates in a fluid of which the resistance varies as the velocity; find the time of a small oscillation.

10. If x, y, z, be the co-ordinates of a particle in a mass of incompressible fluid at any time t, and u, v, w be its velocities in the direction of the co-ordinates at that time; then if

$$udx + vdy + wdz = d\phi,$$

prove that $\frac{d^2\phi}{dx^2} + \frac{d^2\phi}{dy^2} + \frac{d^2\phi}{dz^2} = 0.$

Does this equation occur in any other investigation?

11. If a column of air be confined in a slender cylindrical tube, and the motion of the particles be very small, deduce the equations

$$v = F(x - at) + f(x + at),\ as = F(x - at) - f(x + at)$$

where x is the distance of a small portion from a fixed origin at the end of the time t, $1 + s$ its density, the mean density being 1, and the pressure $= a^2 \times$ density.

Taking the particular value $v = F(x - at) = as$, prove that the velocity with which the motion is propagated is equal to that acquired by falling through half the height of the homogeneous atmosphere.

QUEEN'S COLLEGE, 1825.

1. Compare the pressure on the surface of a sphere filled with water with the weight of a sphere of mercury of the same magnitude.

2. A piece of wood weighs 12 lbs., and when annexed to 20 lbs. of lead and immersed in water, the whole weighs 8 lbs.; required the specific gravity of the wood, that of lead being eleven times that of water.

3. The resistance to a cube moving in a fluid in the direction of its diagonal : resistance to the same cube moving in a direction perpendicular to its side :: 1 : $\sqrt{3}$.

4. The orifices in the equal bases of two upright prismatic vessels are in the ratio of 2 : 1, and the vessels are emptied in equal times; compare their altitudes.

5. A solid of revolution whose axis is perpendicular to the horizon empties itself by a small given orifice in its lowest part; required its nature, when the velocity of the descending surface varies inversely as the ordinate of the generating figure.

6. If the compressive force of the atmosphere varies as the logarithm of the density, and the density varies inversely as the distance; required the law of the force of gravity.

7. In a condenser, if the capacity of the receiver is 30 times that of the barrel, and the gage is 60 inches in length; what part of it will be left free from mercury after 20 strokes of the piston?

8. It was observed that whilst a cylinder of known dimensions discharged $\frac{2}{3}$rds of its contents of water, a pendulum made 125 oscillations; required the length of the pendulum.

QUEEN'S COLLEGE, May 1831.

1. If three fluids, whose magnitudes are as 3, 4, 5, and specific gravities as 2, 3, 4, be mixed together, required the specific gravity of the compound.

2. A cubical vessel is filled with fluid of given specific gravity; find the pressure on its sides.

3. If a given hemisphere containing fluid be whirled round its axis with a given angular velocity and acted on by gravity; compare the pressure on its surface with that when the hemisphere is at rest.

4. A hollow cone of given density just floats in a fluid of given specific gravity with its vertex downwards, find the thickness.

5. Required the greatest density of a fluid in which a given cone will rest in equilibrium of indifference.

6. If a paraboloid with its base downwards and constantly full of fluid be bored with innumerable holes perpendicular to its surface, required the surface which will bound the issuing fluid.

7. Show that the small oscillations of a fluid in a circular tube are isochronous.

QUEEN'S COLLEGE, 1832.

1. If the weights of two fluids be 40, the sum of their magnitudes 15, and their specific gravities 2, 3: required the magnitudes.

2. A parallelogram is immersed vertically in a fluid with one side coinciding with its surface, find the centre of pressure.

3. Determine the time in which a cone filled with fluid with its axis parallel to the horizon, will empty itself through a small orifice at the lowest point of its base.

4. A cone is immersed in a fluid with its axis vertical and vertex downwards; shew that the distance of the metacentre from the centre of gravity of the displaced fluid varies as depth of vertex from surface.

5. If a body acted on by gravity be projected in a given direction and with a given velocity, in a uniform medium, of which the resistance varies as velocity, then the vertical space fallen through by the body which is due to the resistance $\propto$ (abs.)^3 nearly.

6. A hollow triangular prism is filled with fluid and revolves round one of its edges which is vertical with the angular velocity ω; required the pressure on each of its sides.

CORPUS CHRISTI COLLEGE, June 1832.

Third Year.

1. Explain the meaning and extent of the principle which is usually stated thus : "fluids press equally in all directions," and how is the principle established?

2. Define the terms "specific gravity" and "density."

3. Shew that the common surface of two fluids which do not mix is a horizontal plane.

4. What mechanical conditions must be satisfied that a solid may rest in a fluid; and how are they established?

5. How would you determine the nature of the equilibrium of a solid floating in a fluid? Shew how the position of the metacentre may be found. (Be particular in stating the kind of displacement you are investigating.)

6. Investigate the general equation which must be satisfied that a mass of fluid, acted upon by any forces, may be in equilibrium, and shew from the results of analysis that you would not expect the atmosphere near the earth's surface to be at rest.

7. Required the force with which a stream impels a plane, the plane being perpendicular to the direction of the stream. State also the particular suppositions which are made in the usual theory of resistances, and why the results of such a theory and experiment should not be expected to coincide very exactly. I believe the results differ in proportion as the velocity increases. Can you shew any reason why this might be expected?

8. Explain the construction of the hydraulic ram and Bramah's press.

9. If a fluid of any kind be moving in such a manner that at the same point of space the velocity is always the same both in quantity and direction, and X, Y, Z are the forces impressed on any point whose co-ordinates are x, y, z, shew that

$$\frac{dp}{\rho} = Xdx + Ydy + Zdz - vdv.$$

10. A given rectangular parallelogram is immersed in a fluid of given specific gravity with one corner coincident with the surface of the fluid and its side inclined at a given angle to the surface; find the position of the centre of pressure.

11. A given paraboloid is floating in a fluid of given specific gravity when the equilibrium is slightly deranged: define the motion which will ensue. (Be particular in stating the exact kind of motion you intend to investigate.)

12. The consideration of the motion of a slender column of air, influenced only by its own elasticity, gives rise to the equation $\frac{d^2v}{dt^2} = a^2 \frac{d^2v}{dx^2}$ where v = velocity of a particle at a distance (x) from a fixed point at end of time (t). Explain the nature of the motion, and shew that the velocity of propagation of the wave $= a$.

13. By supposing $udx + vdy + wdz$ a complete differential and equal to $d\phi$, the equations for determining the motion of a mass of fluid assume this form, viz.

$$\frac{d^2\phi}{dx^2} + \frac{d^2\phi}{dy^2} + \frac{d^2\phi}{dz^2} = 0, \text{ and } (\text{velocity})^2 = \left(\frac{d\phi}{dx}\right)^2 + \left(\frac{d\phi}{dy}\right)^2 + \left(\frac{d\phi}{dz}\right)^2.$$

What can be determined with regard to the action of the parts of the fluid on each other independently of any supposition with regard to the original disturbance, and what circumstance of the motion is implied in the analytical fact that $udx + vdy + wdz$ is a complete differential?

St. PETER'S COLLEGE, May 1831.

1. In order that a fluid mass may be in perfect equilibrium, shew that we must have

$$dp = \rho X dx + \rho Y dy + \rho Z dz$$
$$\text{and } \rho X dx + \rho Y dy + \rho Z dz,$$

a perfect differential of three independent variables; X, Y, Z being the accelerating forces acting on a particle whose co-ordinates are x, y, z, and ρ the density of the fluid at that point.

If the only forces are the mutual attractions of the particles according to the law of gravitation, and centrifugal force, shew that the second condition will be satisfied.

2. If the density of the fluid $\propto$ depth, find the pressure upon a triangular plane, of which one angle is a right angle, and the base of which coincides with the surface of the fluid, the inclination of the plane to the surface being θ.

Determine θ so that the vertical pressure may be a maximum.

3. When a body floats in a fluid, the weight of the body equals that of the fluid displaced.

4. If a floating body be made to revolve, so as to pass successively through different positions of equilibrium, shew that those of stable and unstable equilibrium occur alternately.

Does the converse of this necessarily hold?

5. If u be the velocity at a horizontal orifice and z the depth of the fluid, prove the formula

$$u^2 = -\frac{2g}{a^2}\,\epsilon^{\int_z \frac{\frac{k^2}{a^2}-1}{kN}} \int_z \frac{kz}{N}\,\epsilon^{-\int_z \frac{\frac{k^2}{a^2}-1}{kN}},$$

a being the area of the orifice, k that of the surface of the fluid, and

$$N = \int_x \frac{1}{X} \text{ from } x = 0, \text{ to } x = z,$$

X being the area of a section of the vessel at the height x.

Will this formula hold for a finite orifice which is not horizontal?

6. Determine the expression for the resistance on the edge of a plane of given thickness, and bounded by a given curve, in terms of the relative velocity of the plane and fluid, and the density of the fluid.

7. Explain the action of the syphon. How does it begin to act? Determine the accelerating force on the fluid within the tube.

8. Shew how the place of a barometer may be supplied by weighing a body of considerable bulk and small specific gravity.

9. If a given globe of very nearly the same specific gravity as water, be placed in a stream running with a given velocity, determine its motion.

10. If a cylindrical vessel containing a given quantity of water, be drawn up vertically by means of a weight and string passing over a pulley, determine the pressure of the fluid on the sides of the vessel.

11. Investigate the following formula for finding the height of a mountain by means of a barometer,

$$z = C\left\{1 + \frac{2(t + t')}{1000}\right\}\left\{\log.\frac{h}{m\left(1 + \frac{T - T'}{5412}\right)h'} + 2\log.\left(1 + \frac{z}{r}\right)\right\}\left(1 + \frac{z}{r}\right),$$

where $\frac{1}{250}$ expresses the expansion of an elastic fluid for 1° of the thermometer, $\frac{1}{5412}$ the condensation of mercury for each degree, and C a constant quantity.

How may the constant C be determined; and what correction is necessary on account of the position of the place on the earth's surface?

JESUS COLLEGE, June 1832.

1. How is the pressure of a fluid at a given point measured? Prove that fluids press equally in all directions—upon what axiom does this depend?

2. The surface of a fluid at rest is a horizontal plane.

3. Find the pressure of a fluid on any surface, and thence find the pressure of a fluid whose density $\propto$ (depth)^n on a surface of revolution whose axis is vertical.

4. Find the centre of pressure on any plane surface.

5. Shew in what case the equilibrium of a solid in a fluid is stable or unstable. Define the metacentre and find its position.

6. Find the pressure at any point in a mass of fluid at rest, acted on by any forces: and shew that the density is the same at all points of the surface.

7. Find the resistance to a solid of revolution, and apply it to find the resistance on a sphere.

8. Compare the specific gravities of a solid and fluid by weighing the solid in air and in the fluid.

9. Explain the method of filling a common thermometer, and compare the scales of two that are differently graduated.

10. Find the least play of the piston which will enable a common pump to work.

11. Find the height of a mountain by means of a barometer and thermometer.

12. Find the velocity of a fluid issuing through a finite orifice.

SIDNEY SUSSEX COLLEGE, May 1829.

1. Define the specific gravity of a body: shew that it varies directly as the weight and inversely as the magnitude. What are the standard weight and magnitude assumed in the equation $S = \frac{W}{M}$?

2. If a surface be immersed in a fluid at rest, the perpendicular pressure upon it is equal to the weight of a column of the fluid, whose base is the area of the surface, and altitude the perpendicular depth of its centre of gravity.

Example. Let a hollow sphere be filled with water: divide it by a circle parallel to the horizon into two parts which shall be equally pressed.

3. Find the positions of equilibrium of an homogeneous triangle, floating on a fluid with one angle immersed.

4. When a fluid issues from a small orifice in the bottom or side of a vessel, explain the formation and effect of the vena contracta.

5. Determine the relation between the time and the quantity of fluid issuing from a vessel through a vertical orifice, the vessel being kept constantly full.

Example. Let the orifice be a circle, and the surface of the fluid a tangent to its upper extremity.

6. If a plane be opposed obliquely to a stream, find the force of the stream in the direction of its own motion, and in the direction perpendicular to it.

7. If a fluid consist of particles which repel each other with forces varying inversely as d^n, d being the distance between their centres, find the law of the compressing force in terms of d, and in terms of the density.

8. The form and dimensions of a diving-bell being given, investigate the relation between the depth to which it has sunk, and the height to which the water has risen within it.

Example. Let the bell be a hemisphere of given radius (6 feet), and let the surface of the water within it bisect its vertical radius; the barometer standing at 30 inches, find the depth of the water.

9. Explain the construction and action of the fire-engine.

10. Let a given cylindrical rod float vertically on a fluid, whose specific gravity is known: if the whole be placed under the receiver of an air-pump, find the change in the depth to which the rod will sink at each turn of the handle; and its limiting depth when the number of turns is increased indefinitely.

11. Let two cylinders of given capacities contain airs of different densities: a communication being opened between them by a small orifice, find the time in which the densities will become equal.

SIDNEY SUSSEX COLLEGE, May 1830.

1. Define a fluid; what is an incompressible, and what an elastic fluid? explain the term "perfect fluidity."

2. Define "specific gravity," and show that in any substance the volume multiplied into the specific gravity, is equal to the weight.

3. When the particles of a fluid mass in equilibrium are acted on by any number of forces, find an equation for determining the pressure at any point; explain accurately how this pressure is estimated; and show in what manner the expression is to be applied in finding the pressure on any surface immersed in a fluid.

4. A hollow sphere is filled with fluid and whirled round with a given angular velocity, the particles of fluid being attracted towards the centre of the sphere by a force varying as the distance; find the pressure on the surface of the sphere.

5. One end of a regular trough is an isosceles triangle, moveable about a hinge at the vertex, when the trough is filled with fluid, find the force that must be applied at the centre of gravity of the triangle to keep it from moving.

6. Explain the construction and use of Nicholson's hydrometer, and give the method of determining the specific gravity of a body lighter than the fluid in which it is weighed.

7. If a body float on a fluid, the weight of the fluid displaced is equal to the weight of the body, and the centres of gravity of the body and fluid displaced are in the same vertical line.

8. If a vessel be constantly filled with fluid, and a very small orifice be made in the side or bottom of the vessel, the velocity of the issuing fluid is nearly that due to the depth of the orifice.

9. Find the time in which a given cone will empty itself by a small orifice at the vertex, the slant side being placed parallel to the horizon.

10. Find the resistance on the surface of a solid of revolution moving in a fluid in the direction of its axis. How is the hypothesis incorrect on which the laws of this resistance are founded?

11. Compare the resistances on a cone and paraboloid of equal bases, moving in a fluid with equal velocities in the direction of their axes.

12. Explain the action of the common pump, and find the height to which the water rises after (n) strokes of the piston.

13. Show how the heights of mountains may be determined by means of the barometer and thermometer, and explain how to correct for the variations in the latitude, and in the temperature at the top and bottom of the mountain.

OPTICS.

TRINITY COLLEGE, 1822.

1. If a plane mirror be made to move in a conic section, so as always to touch it, find the path described by the image of an object placed in the focus.

2. Parallel rays may be made to converge much more nearly to the same point by means of a reflector generated by the revolution of a small arc of a catenary round its axis, than by a spherical reflector of the same dimensions.

3. If F be the focal length of a spherical reflector, or refractor, and D the distance from the centre of a straight line perpendicular to its axis, then the polar equation of the image is $r = \dfrac{F}{1 - \dfrac{F}{D} \cdot \cos.\theta}$. Prove this, and find the axes of the conic section to which it belongs.

4. If a ray of light be refracted through any number of mediums contained by parallel plane surfaces, it will be as much bent from its original course as if it passed immediately out of the first medium into the last.

5. When a ray of homogeneal light is incident obliquely upon a spherical refracting surface, determine the intersection of the refracted ray with the axis of the pencil to which it belongs.

6. If the focus of rays incident on a convex lens of inconsiderable thickness be near its axis, then the focus of refracted rays may be determined from the formula $\frac{1}{d} + \frac{1}{f} = \frac{1}{F}$, where d, f, F denote the distances from the centre of the lens of the foci of incident, and refracted rays, and of the principal focus respectively. Prove this, and shew how the formula may be applied to all the other lenses.

7. Supposing the sun's rays, after being refracted by the earth's atmosphere, to pass through it in right lines, and no reflection to take place; find the dimensions of the illumined part of the atmosphere which is opposite to the sun.

8. The density of rays in the sun's image formed by a reflector

$$\propto \frac{\text{area of the aperture} \times \text{reflecting power}}{(\text{focal length of the reflector})^2}.$$

9. If rays be incident parallel to its axis on the plane surface of a plano-convex lens, whose thickness $= t$, and radius $= r$; and emerge after two refractions at the plane surface, and one reflection from the spherical; prove that the distance of the geometrical focus of the reflecto-refracted rays from the plane surface $= \frac{r - 2t}{2n}$; where $n = \frac{\text{sin. incidence}}{\text{sin. refraction}}$.

10. Construct Galileo's telescope, find its magnifying power, its greatest field of view, and shew how it must be adjusted to the eye of a short sighted-person.

11. Describe the experimentum crucis, by which Newton shewed that the primary colors cannot be separated into others by refraction.

12. If $n = \frac{\text{sin. incidence}}{\text{sin. refraction}}$ from air into glass for the rays of mean refrangibility, and $n \pm \delta n$ denote the same for the greatest and least refrangible rays; also if $\phi =$ angle at which the sun's rays are incident on an isosceles prism whose angle $= 2\alpha$, then the angle contained between the rays of greatest and least refrangibility $= \frac{4 \sin.\alpha \,.\, \delta n}{\cos.\phi}$.

13. In a convex lens with surfaces of equal radii, the spherical aberration will exceed the chromatic, if the semi-aperture of the lens be greater than $\frac{1}{9}$ of its radius.

14. If the dispersive powers of two prisms placed one against the other in opposite directions, be inversely as their refracting angles; a ray of light incident nearly perpendicularly on either prism, and refracted through both will emerge free from color.

15. Explain fully the formation of the primary and secondary rainbows, and find their altitudes and breadths.

16. (1). If a tangent be drawn to the generating circle of a cycloid at its generating point, and be taken equal to the ordinate of the circle at that point, prove that the extremity of the tangent will trace out the caustic formed by rays incident on the cycloid in a direction parallel to the base.

(2). Find the length of this caustic, its highest point, its point of regression, and the area contained between the cycloid, the caustic, and the reflected ray.

17. The caustic by refraction of a plane surface, when rays diverge from a point, is the evolute of an ellipse, or an hyperbola, according as the rays pass from a denser into a rarer, or from a rarer into a denser medium.

18. If the quantity of light emitted by any particle of a luminous spherical superficies towards a point placed within it, be supposed to vary as the sine of the angle, which the emitted rays make with the surface: then the point will receive the same quantity of light, whatever be its position, and whatever be the magnitude of the superficies.

TRINITY COLLEGE, 1824.

1. What are the two principal theories which have been formed on the nature and propagation of light? Would the mathematical explanation of the common phenomena of reflected and refracted light be the same upon both hypotheses?

2. If Δ and Δ' be the distances of the foci of incident and reflected rays from the surface of a spherical reflector, whose radius is r, then

$$\frac{1}{\Delta} + \frac{1}{\Delta'} = \frac{2}{r},$$

the direction of incidence being nearly perpendicular to the surface.

3. The reflecting curve is a circle, and the radiating point is the extremity of the diameter: to describe the caustic.

4. What is the angle at which two plane reflectors must be placed with respect to each other, so that the images of an object placed between them may be found in the angles of an equilateral pentagon?

5. If rays are reflected at the back of a plane looking-glass, whose thickness is given, given the focus of incident rays, to find the focus of emergent rays, the direction of incidence being nearly perpendicular to the surface.

6. Find the deviation of a ray passing through a prism, whose refracting angle is inconsiderable.

7. The conjugate foci in a spherical refractor move in the same direction upon the axis and coincide at its surface and centre.

8. Find the focal length of a double convex lens.

9. The power of a compound lens is equal to the sum of the powers of the component lenses, the power of a lens being defined to be the reciprocal of its focal length.

10. Find the focal length of a double concave lens by experiment.

11. Newton, in describing the experiment with the prism for determining the unequal refrangibility of light, says, that the image of the sun, whilst the prism was turned, first descended, and then ascended, and for one position was stationary; in that position, the refraction of the light at the two sides of the refracting angle of the prism was the same: prove this, and shew the use of this circumstance in the experiment in question.

12. In a double achromatic object-glass, the powers of the lenses must be inversely as the dispersive powers of the glass of which they are respectively formed.

13. A short-sighted person can see distinctly at the distance of six inches: what must be the power of a lens to enable him to see distinctly at the distance of 18 inches?

14. In the common Astronomical telescope, given the focal lengths and diameters of the object-glass and eye-glass, find expressions for its magnifying power, field of view, and for the brightness of the image compared with that of the object.

15. Explain the construction of the Gregorian telescope, and find its magnifying power.

16. The colours of the secondary rainbow occur in the inverse order of those in the primary.

17. Find the longitudinal and lateral aberrations of parallel rays incident upon a spherical refractor.

18. Find the centre and diameter of the least circle of chromatic aberration in a lens, and compare the density of the rays in different parts of it.

19. Find, from the requisite data, the time which an equatoreal star is in passing over the field of view of a Newtonian telescope.

20. A person sees distinctly in air: what must be the nature and power of the lens which he must use in order to see distinctly under water?

21. Upon what physical principle has Newton explained the ordinary refraction of light? Are the magnitudes of the forces which are requisite in order to reconcile the phenomena of refraction with his hypothesis, any objection to its truth?

22. State generally the phenomena of double refraction which are observable in crystals with one axis.

TRINITY COLLEGE, MAY 1831.

1. A SMALL pencil of parallel rays falls directly on a concave spherical mirror; determine the longitudinal aberration of the extreme ray.

2. In the direct incidence of a small pencil of rays on a spherical mirror, $QF : FA :: FA : Fq$. Prove this for all the different cases, and shew that when the incident pencil is convergent to a point behind a concave mirror, the convergence of the reflected pencil will be increased by a constant quantity.

3. Explain fully the nature of vision to an eye under water.

4. A small pencil of rays is refracted obliquely at a plane surface, if QH (u) be the axis of the incident pencil, and Q', Q'', the foci of the refracted pencil in the primary and secondary planes, shew that

$$Q'H = \mu u \frac{\cos.^2 \phi'}{\cos.^2 \phi}, \text{ and } Q''H = \mu u.$$

5. Investigate a formula for the refraction of a pencil of rays incident directly on a concave spherical surface, giving the first and second approximations.

6. Having obtained a formula for the refraction of a pencil of diverging rays proceeding out of a rarer into a denser medium, how is it to be adapted to rays proceeding out of a denser into a rarer?

Exemplify this in determining q_1 and q_2 the foci (after the second refraction) of an oblique pencil incident on a plate with parallel surfaces, referring to question (4).

7. The deviation of a ray of light is always from the refracting angle of a prism, if it be denser than the surrounding medium.

8. Find the minimum distance of the conjugate foci Q and q, when on different sides of a lens. How does this enable us to determine the focal length of a lens by experiment?

9. Four convex lenses having a common axis are placed at intervals m, $5m$, $10m$ from each other, the focal lengths of each of the three first being $5m$, and of the last m. If a pencil of parallel rays fall on the first, determine the point to which they converge after passing through the system.

10. Diverging rays are incident directly on a refracting sphere; determine the focus of refracted rays.

11. State the experiment by which Newton shewed that the sun's light consists of rays which differ in refrangibility and colour, and that each elementary ray of the solar spectrum when once separated does not admit of further separation by another refraction. Describe the appearance of the spectrum when in its highest state of purity.

12. What is meant by the dispersive power of a medium, and the irrationality of the dispersions of different media?

13. Determine the relation between the focal lengths of two lenses which shall achromatize each other, when placed in contact.

14. Explain the formation of the rainbow, and why the colours of the primary and secondary bow are in a reverse order.

15. Describe the astronomical telescope in its simplest form, drawing the course of the extreme rays. Determine its field of view. How is a uniformity of brightness secured in it? Why

cannot that be effected in the Galilean construction, as used in practice?

16. Construct the Gregorian telescope, and determine its magnifying power, tracing the course of a pencil of rays.

17. When a pencil of rays passes eccentrically through a lens, explain the nature of the confusion arising from the chromatic dispersion. How is that remedied? Determine the intervals between two lenses of given focal lengths, so that the directions of all the partial emergent pencils may be parallel, the axis of the incident pencil being parallel to that of the lens.

18. What are the requisites (respecting the object-glass and eye-piece) that vision through a telescope may be as distinct as possible?

19. Prove the law of reflexion of light on the theory of undulations.

20. What is the principle of the interference of rays of light? How is the existence of such interference demonstrated?

21. State the phenomenon of double refraction, and the properties of the two refracted pencils.

TRINITY COLLEGE, June 1832.

1. State the known general properties of light, and explain the terms reflexion, refraction, and unequal refrangibility.

2. Explain the difference between the least circle of aberration, and the least circle of chromatic dispersion.

3. A small pencil of rays, diverging from a point on the axis of a mirror, is reflected at the centre of the mirror, find the focus of reflected rays; and shew that the conjugate foci are always on the same side of the principal focus.

4. Find the position and diameter of the least circle of aberration, when the radiating point is in the axis of a given concave spherical mirror.

5. A small object is equidistant from two plane parallel mirrors, find the path of the axis of the small pencil of rays, by which an eye

placed in a given position between the mirrors, can see the third image proceeding from either side, and shew that its length equals the distance of the eye from the image.

6. In the direct incidence of a small pencil of rays upon a spherical refracting surface, shew that the conjugate foci move in the same direction, and are coincident at the surface and centre of the refractor.

7. A small pencil of rays is refracted obliquely at a definite point on a spherical surface, find the distances from that point of the foci of refracted rays in the primary and secondary planes.

Adapt the general expression for the primary plane, to the case where the obliquity is small, but not wholly inconsiderable.

8. When a centrical pencil of rays is refracted at a convex spherical surface, find the aberration, and shew that there will be no aberration when $QA = (\mu + 1)r$.

9. If a small pencil of rays enters a lens directly, and is emergent after reflection at the second surface, the effect upon the pencil is that of two passages through the lens and a reflection at the second surface, the thickness of the lens being neglected.

10. Find the focal length of the sphere considered as a lens.

11. Find the place of a double concave lens when its conjugate foci are given in position.

12. When a pencil of rays passes through a given prism, and both refractions take place in the same plane, and very near to the refracting angle, the emergent rays will diverge accurately from a point, if the angles of incidence and emergence be equal.

13. Investigate an expression for the dispersive power of a medium, and find the condition requisite, that two prisms placed contiguously with small refracting angles may achromatise each other.

14. Find the direction under which a small pencil of rays must enter a sphere of water, in order that the rays in the primary plane may emerge parallel after (p) reflections within the drop.

15. Explain the formation of the rainbow, and find the altitude of its highest point, and the breadth of the colours.

16. Find the magnifying power of a simple microscope, whose focal length is $\frac{1}{10}$ inch, and its distance from the eye $\frac{1}{4}$ inch, the least distance of distinct vision being 12 inches.

17. Describe the Newtonian telescope, and find its magnifying power and field of view.

How must this telescope be adjusted to the eye of a short-sighted person?

18. In the undulatory theory, explain the terms, wave, length of a wave, phase of a wave, and front of a wave, and state the principle by which the disturbing effect of a wave may be calculated.

19. Explain the refraction of light on the undulatory theory.

20. State the principal phenomena of polarized light, and the hypothesis suggested for their explanation.

St. JOHN'S COLLEGE, 1815.

1. Explain the experiment by which it appears that the sine of incidence : sin. refraction in a given ratio.

2. The distance of P from the circumference of a polished circle is equal to that of Q from its centre. If $PABQ$ be the course of a ray reflected at A and B, then

$$AB - AP : AB - BQ :: BQ : AP.$$

3. A very small polished angular solid receives upon two of its adjacent planes respectively the light of two candles, so situated as both to become visible at once by reflection to an eye placed at a considerable distance. The angle contained between the reflecting planes is equal to half the angle between the directions of the incident rays.

4. A speck in the middle of the back of an isosceles prism will appear double to an eye placed close to its edge. Suppose the angle which the two images so seen subtend, at the eye, to be a right angle, determine the angle of the prism, and shew that the supposition is impossible, unless sin. inc. : sin. ref. in a greater ratio than $3 + 2\sqrt{2} : 1$. out of air into the prism.

5. A ray of light emitted from a luminous point reaches the eye after reflection at a convex polished surface. The position of the eye and radiant point being given, prove, by a property of the ellipse, that the path described by the ray is the shortest which could be passed over by a body, so as to reach the eye after meeting the surface. Explain why this is not universally true in the case of a concave surface.

6. A book, laid horizontally on a table, is illuminated by a candle placed at a certain distance from it. As the candle burns equably down, determine at what height of its flame the illumination of the page will be the greatest.

7. A parabolic reflector is employed to view near objects. Prove that the aberration of extreme rays $= \frac{(\text{semi-aperture})^2}{4QS}$, QS being the distance of the object from the focus; and determine whether the focus of extreme rays will be nearer to or further from the vertex than that of central rays.

8. When a prism, not very acute-angled, is laid on its base before an open window and the eye is situated at a proper angle of elevation above the base, a circular bow of a blue colour, concave towards the eye, is seen in the base, forming the boundary of light and darkness. Explain the cause of this phenomenon.

9. Find the focal length of a lens whose thickness is not inconsiderable, and shew that if R, r, be the radii of the first and second surface, $t = \frac{\text{thickness}}{R + r}$, and $1 + \mu : 1 :: \sin. \text{inc.} : \sin. \text{ref.}$, the focal length will be $\left(\frac{1 + \mu}{\mu}\right) \cdot \frac{R \cdot r}{R + r} \cdot \frac{1}{1 + \mu(1 - t)}$, and the distance between the focal centres $= (R + r) \cdot \frac{\mu(1 - t)}{1 + \mu(1 - t)} \cdot t$.

10. A spectator at sun-set sees a rainbow from the top of a very high mountain, and observes that it exceeds a semi-circle by a certain arc. How may the height of his station be determined?

11. A ray of light is incident on a plane. Given the equation of the plane, and also that of the line described by the ray before incidence, it is required to determine the equation of the ray; 1st,

after reflection; 2ndly, after refraction at the plane. In the latter case apply your result to find the caustic surface of a plane.

12. Shew that the deviation of a ray refracted through a prism denser than the surrounding medium is towards the thicker part of the prism.

13. Describe the phænomenon of double refraction, and the law which the extraordinary ray observes.

14. Given the positions of a radiant point, and the eye, in a line nearly perpendicular to the side of a very acute-angled prism; determine the course of a ray which shall reach the eye after refraction.

St. JOHN'S COLLEGE, Dec. 1816.

1. Define the geometrical focus of a pencil of rays; and shew, that in the case of a spherical reflector that point is farther from its surface than the intersection of any oblique reflected ray with the axis of the pencil.

2. Shew that if an object be viewed through a medium contained by parallel plane surfaces, the aberration of oblique rays from the geometrical focus is less than when the rays are refracted at a single surface.

3. If two equal and parallel pieces of mirror-glass containing a thin plate of air between them be closely united and immersed in a vessel of water, an object at the bottom can be plainly seen through them, until the angle at which they are inclined to the surface becomes considerable, after which the object cannot be seen. Required an explanation of this phænomenon.

4. If rays fall nearly perpendicularly on a spherical refracting surface, shew that QF and fq are always measured in opposite directions from F and f.

5. Find the focal length of a meniscus of small thickness, and shew that it is not altered by inclining the incident pencil a little to the axis of the lens.

6. A circle is immersed vertically in water. Required the figure and dimensions of the image. Find the same when it is seen through a medium contained by parallel plane surfaces.

7. Compare the burning powers of a spherical reflector and double convex lens of equal aperture, the radii of whose surfaces are equal and double that of the reflector, the reflecting power of the one and transmitting power of the other being supposed equal.

8. Explain the construction and effect of the multiplying glass.

9. Point out the respective advantages of Galileo's and the common astronomical telescope, and find where the eye must be placed so that the field of view in the latter may be greatest.

10. Investigate the form of the surface that shall accurately reflect parallel rays.

11. Find the least circle into which a pencil of rays reflected by a spherical surface is collected, and shew that when the incident rays are parallel the radius of this circle $\propto \frac{(\text{semi-aperture})^3}{(\text{focal length})^2}$

12. The reflecting curve is the common parabola, rays incident parallel to the ordinates; find the nature of the caustic, the density of rays in different points, and its length for any portion of the parabola.

St. JOHN'S COLLEGE, Dec. 1821.

1. Shew that it is not necessary to constant vision, that rays of light should consist of contiguous particles.

2. Explain the principle and construction of a common opera glass.

3. Two parallel rays are incident on a spherical reflector on the same side of the axis; shew that the angle between the reflected rays is equal to twice the difference between the angles of incidence.

4. The distance at which a short-sighted person can see distinctly is 3 feet. He has a double concave lens, whose focal length is 3 feet. Will this enable him to see an object at the distance of 12 feet? if not, find the nature and focal length of a lens, which placed between his eye and the former glass will be sufficient for that purpose.

5. When a small pencil of homogeneal rays falls obliquely upon a plane refracting surface, and in a plane which is perpendicular to that

surface, having given the focus of incident rays, and the angles of incidence and refraction, find the geometrical focus of refracted rays.

6. *ABC* is an equilateral triangle, *PQRSTV* the course of a ray refracted at *Q* and *T*, and reflected at *R* and *S*. The angle of incidence of *PQ* is 15°; find the ratio of sin. *I* : sin. *R*, that the incident and emergent rays may be inclined at an angle of 30°, and shew that in that case *QRST* cannot be greater than 2 *AB*, nor less than *AB* at whatever point in *AB* the ray is incident.

7. *A* and *B* are two fixed points, and *CD* is a plane reflector which moves parallel to itself. A ray of light proceeding from *A* is reflected to *B* in every position of the reflector; find the locus of the points of incidence.

8. When diverging rays are incident on a rarer medium contained by parallel plane surfaces, the geometrical focus is farther from the surface than the focus of incident rays.

9. *BAD*, *BCE* are two plane reflectors inclined to each other at an angle of 15°. *A* is a given luminous point in one of them. Find at what angle a ray proceeding from *A* must be incident on the other reflector, that after 3 reflections it may be parallel to *BA*; and if *ACDE* be a small pencil, compare the densities at *C*, *D*, *E*, supposing no light absorbed by the reflectors.

10. Construct a Newtonian telescope, and determine the field of view.

11. If parallel rays be incident on a spherical reflector, whose aperture is small, prove that the longitudinal aberration varies as the square, and the lateral aberration as the cube of the diameter of the aperture.

12. In the experiment where Newton shews that the Sun's light consists of rays differing in refrangibility and colour, prove that when the spectrum appears stationary, the angles of refraction at each side of the prism are equal.

St. JOHN'S COLLEGE, Dec. 1822.

1. A convex lens increases the convergency and diminishes the divergency of the rays of any pencil incident upon it.

2. A ray of light issuing from a point between two plane reflectors, and falling on one of them at a given angle, after (n) reflections becomes parallel to the reflector on which it was first incident. Required the angle at which the reflectors are inclined.

3. Required the image of an object placed before a prism of small vertical angle.

4. Place a straight line before a spherical reflector,

(1). So that it may pass through the centre of the Conic Section which forms the image.

(2). So that the image may be a circle; or a rectangular hyperbola.

5. When a luminous particle is placed before a spherical refractor, find the position of its image. Find also the situation of the particle, so that its distance from the image may be a minimum.

6. A glass sphere and a double convex lens have the same focal length, and the radius of one of the surfaces of the lens is twice that of the other. Compare the radii of the sphere and lens.

7. In the diameter of a given circle, two lights are placed, whose distances from each other is equal to the radius; their distances from the centre being as 3 : 1. Supposing the intensities of the lights to be inversely proportional to their distances from the centre of the circle, determine those points in the circumference which receive the least light possible.

8. Compare the fields of view in the Telescopes of Gregory and Cassegrain, the reflectors and eye-glass being supposed the same in each. Point out the respective advantages of the two instruments.

9. Having given the focal length (α) of a spherical refracting surface, and (β) that of a thin convex lens; find what must be the focal length of another lens, to be compounded with the former, such that the refractor and compound lens may, with equal apertures, form equally bright images of distant objects: supposing the transmitting power of the refractor to be to that of the lens :: n : m.

10. Find the nature and magnitude of the image of a straight line placed before an hyperbolic speculum, at a given distance from one of its foci; the eye being situated in the other focus.

St. JOHN'S COLLEGE, Dec. 1830.

1. If a ray of light be reflected once by each of two plane mirrors; and in a plane which is perpendicular to their common intersection, the angle contained between the first and last directions of the ray, is equal to twice the angle at which the mirrors are inclined to each other.

2. Parallel rays may be made to converge or diverge accurately by means of a parabolic reflector.

3. Having given the inclination of two plane mirrors, and the situation of an object between them; find the number and position of the successive images formed by them.

4. A ray of light, which passes through a prism in a plane perpendicular to its edge, is turned towards the thicker part, or from it, according as the prism is denser or rarer than the surrounding medium.

5. Find the refracting power of any transparent substance.

6. Find the longitudinal aberration, when a pencil of rays is incident, at a small but finite angle, upon a plane refracting surface.

7. Find the focal length of a given sphere.

8. Define the centre of a lens, and determine its position in a double convex lens of given thickness.

9. Find the visual angle of a given object, when viewed through a double convex lens.

10. Construct Sir I. Newton's telescope, determine its magnifying power, and shew how it may be adapted to nearer objects.

11. Explain the phenomenon of the unequal refrangibility of light; and find the dispersive power of the substance of which a prism, having a very small refracting angle, is formed.

12. When a beam of solar light falls upon a lens, find the centre and diameter of the least circle of chromatic aberration.

13. Explain the reason why the order of the colours in the primary and secondary rainbows, is inverted.

14. Having given the radius of an arc of any colour in the primary rainbow, find the ratio of refraction, for that colour, out of air into water.

St. JOHN'S COLLEGE, May 1831.

1. When diverging or converging rays are incident nearly perpendicularly upon a lens, the thickness of which is inconsiderable, prove that $QF : QE :: QE : Qq$.

2. If a ray QA be incident upon a spherical reflector, the centre of which is E, in a direction parallel to its axis EC, and AT be a tangent at A cutting the axis in T, shew that the longitudinal aberration $= \frac{1}{2} CT$.

3. Having given the distance of a luminous point from a concave spherical mirror, find its radius, when the aberration of a ray incident at an angle of 45° : the distance of the geometrical focus from the centre :: the distance of the focus of incidence from the centre : the radius of the mirror.

4. Find the illumination of a globe resting on a horizontal plane at a given distance from a candle; the height of the flame of the candle being equal to the radius of the globe.

5. At the bottom of an empty spherical basin a crown piece is placed, and an eye is so situated as just to see the edge of the crown piece over the rim of the basin. When the basin is filled with water the whole crown piece becomes exactly visible. Find the radius of the basin.

6. Find the equation to the caustic, when the reflecting curve is the rectangular hyperbola, and the rays are incident parallel to its axis.

7. If a ray of light be incident upon a prism, in a plane perpendicular to its edge, at an angle of 45°; find the value of the refracting angle of the prism, so that the ray may just emerge parallel to the second surface; the value of the index of refraction (μ) being $\sqrt{2}$.

8. A given circular object is placed before a glass prism, the refracting angle of which is 30°, with its plane parallel to the face of

the prism. Shew that the image is an ellipse, in which the squares of the axes are as 31 : 36.

9. Having given the linear magnitude of a concave spherical mirror of 6 inches radius, before which a small object is placed at a distance of 12 inches; find the linear magnitude of a double convex glass lens, of the same radius, which will form an equally bright image of the object, supposing no rays to be lost by reflection or refraction.

10. The magnifying power of a common opera glass, (or Galilean Telescope,) when directed to a distant object, is 4; but when adjusted to an object situated at a distance of 40 feet from the object glass, the magnifying power is 5. Find the focal lengths of the object and eye glasses.

11. A ray of light diverging from a luminous point is incident upon a plate of glass, the thickness of which is (t), at an angle

$$\left(\tan.^{-1} \frac{4}{\sqrt{5}}\right).$$

Shew that the longitudinal aberration is accurately $\doteq \frac{4}{15}t$.

12. Determine the relation between the focal lengths of two lenses, which will achromatize each other, when separated by a given interval (a).

St. JOHN'S COLLEGE, May 1832.

1. Describe Hadley's quadrant, and find the error in an angle observed with it when the index-glass is not quite perpendicular to the plane of the instrument.

2. If a ray of light be reflected once by each of two plane surfaces, in any planes, the first and last directions of the ray make equal angles with the intersection of the reflectors; and if planes be drawn through the intersection of the reflectors and the first and last directions of the ray, the angle between these planes is equal to twice the angle between the reflectors.

3. QR is a ray incident upon a reflecting surface at the point R, ER is a normal to the surface, and Rq the reflected ray; ER

makes with each of three rectangular axes angles whose cosines are l, m, n; QR and qR make with the same axes angles whose cosines are a, b, c, and a_1, b_1, c_1: prove that

$$a + a_1 = 2(al + bm + cn)\,l,$$
$$b + b_1 = 2(al + bm + cn)\,m,$$
$$c + c_1 = 2(al + bm + cn)\,n.$$

4. Rays are incident on a surface of revolution in a direction parallel to its axis; its caustic by reflection is a sphere: find the equation to the generating curve of the surface.

5. Find the distance between the foci of incident and emergent rays, when a pencil passes nearly perpendicularly through a refracting medium bounded by concentric spherical surfaces.

6. An astronomical telescope is filled with fluid: having given the refractive powers of the fluid and of the lenses, and the radii of the surfaces of the lenses, find the distance between their centres.

7. Rays incident nearly perpendicularly upon one surface of a double concave lens converge, after reflection, to a point four inches from the lens; when incident upon the other surface they converge to a point five inches from the lens; the deviation of a ray passing close to the rim of the lens is 1°. 43′, and the diameter of the lens half an inch: find the index of refraction of the substance of which it is formed.

8. Find the least distance between the conjugate foci of a very thick lens.

9. If the greatest deviation of a ray refracted through a prism $= D$, the angle of the prism $= I$, its index of refraction $= \mu$,

$$\cos. I - \cos.(D + I) = \sqrt{(\mu^2 - 1)}\cos. I.$$

10. Determine the form which a given pencil of rays will assume on emerging from a prism: both refractions taking place in one plane.

11. Investigate the condition of achromatism of two lenses for an excentrical pencil, when the axis of the incident pencil is inclined to that of the lenses.

12. Explain the refraction of light on the undulatory theory.

13. Explain on mechanical principles the transmission of a wave in which the vibrations are transverse to the direction of the motion.

14. Investigate the law of double refraction in uniaxal crystals.

CAIUS COLLEGE, MAY 1831.

1. THE intensity of light emanating from any point, varies inversely as the square of the distance from that point.

2. Find the equations of those curves, which possess the property of reflecting light accurately, from one given point to another.

3. Determine the lateral aberration in a spherical reflector, of which the aperture is small compared with the focal length.

4. Prove that the length of the caustic by reflexion, is equal to the difference between the entire extreme rays.

5. When a small portion of a curve is placed at a given point, with its radius of curvature directly opposed to that of a given reflector; prove that the difference between the curvatures of the image and object is constant.

6. What is the minimum deviation of a ray of light, passing through a prism of given refracting power?

7. To find the focus of a lens of any thickness.

8. Required the position, and radius of the least circle of aberration in a lens.

9. Describe the Cassegrainian telescope, and find its magnifying power.

10. Determine the radius of the bow, formed by light refracted, after p reflexions within the drops of falling rain.

11. Describe some of the experiments, which prove that in biaxal crystals, neither of the refracted rays follows the Cartesian law of refraction.

12. Find the resultant of two similar interfering rays, on the undulatory hypothesis; and give a construction for determining its amplitude.

CAIUS COLLEGE, June 1832.

1. Define a luminous pencil, and state the principal mod tions which it undergoes at the surfaces of transparent bodies.

2. Describe the experimental proof of the equality of the a of incidence and reflection, and apply this principle to the cons tion of Wollaston's Goniometer and Hadley's Sextant.

3. Trace the corresponding motions of the conjugate foci b flection in a spherical mirror.

4. When light issuing from a given point is reflected by a rical surface in a plane, inclined at a given angle to the pri plane, determine the ultimate intersection after reflection.

5. Determine the number of images, when a body is place tween two inclined plane mirrors.

6. Find the caustic surface by reflection when the radiating is in the periphery of the reflecting sphere.

7. Required the minimum deviation when a ray of light through a prism of given refracting power.

8. Describe Herschel's Telescope.

9. Find the radius and position of the least circle of aberrat a lens, the incident rays being parallel.

10. Explain distinctly the formation of the rainbow, and fin radius of the primary bow.

11. Prove that the quantity of light polarized in reflection, the position of the surface is such that the polarization is com equals the transmitted light polarized by refraction in a plane pendicular to that of incidence.

QUEEN'S COLLEGE, 1825.

1. A Ray of light passes from a point situated at the extr of the diameter of a semicircle, and is reflected by the circumfer determine the point of incidence, so that after reflection the ray pass through a given point in the diameter produced.

2. Suppose the focal lengths of two double convex lenses to be to each other as (m) to (n), and the radii of their first surfaces to be equal; required the ratio of the radii of their second surfaces.

3. A sphere of glass and another of water being placed in air, what must be the proportion of their radii, that their magnifying powers may be the same?

4. It is required to find the principal focus of a concavo-convex lens of a rarer medium, whose thickness is inconsiderable.

5. Required the form and focus of a glass lens which would enable the eye to see distinctly under water.

6. In Gregory's telescope, the focal length of the larger reflector, the position and focal length of the eye-glass, and the distance between the two images of a remote object being given; required to find the position and focal length of the smaller reflector, which will cause the telescope to magnify the object in any proposed ratio.

7. If parallel rays fall upon a single thin lens of a given substance, determine the diameter of the least circle into which all the rays of different colours are collected, the linear aperture of the lens being known.

8. Given the highest point of the under arc of a primary rainbow; required the altitude of the Sun's centre.

9. Suppose the reflecting curve to be a cycloid, and the incident rays to be perpendicular to the axis; required the nature of the caustic.

QUEEN'S COLLEGE, MAY 1831.

1. REQUIRED the distance of the geometric focus of a ray reflected at a spherical surface, the distance of the luminous point being 5 feet and radius of mirror 2 feet.

2. Two plane mirrors are inclined to one another at an angle of 15°, required the number of images of an object placed between them.

3. If r be the radius of a spherical refractor, the lateral aberration for parallel rays $= r \sin.\theta \text{ versin.} \frac{\theta}{m}$ nearly.

4. In the interior rainbow the tangent of the angle of incidence is equal to twice the tangent of the angle of refraction.

5. Required the focus of refraction of a concave and convex lens separated by a given interval.

6. Explain the primary and secondary focal lines of an oblique pencil of rays reflected at a spherical surface, and shew that they are at right angles to one another.

7. If the reflecting curve be an arc of 60°, shew that if a luminous body be placed at one extremity the (density)^2 of light at the points of trisection of the caustic are as the numbers $\frac{6^2}{6^2-1}, \frac{3^2}{3^2-1}, \frac{2^2}{2^2-1}$.

QUEEN'S COLLEGE, 1832.

1. If the distance of a luminous point be 7 feet from a spherical reflector whose radius is 1 foot: find the velocity of the focus of the reflected ray, when the luminous point approaches the reflector at the rate of 2 feet per second.

2. If a light be placed in the centre of an ellipse; the sum of the illuminations of every two points in the circumference at 90° distant from each other will be the same.

3. If a ray of light be refracted through a right-angled triangular prism in a plane perpendicular to the axis; then having given the angle of incidence ϕ and the deviation δ, we may find the angle of refraction ϕ' at the first surface; from the equation

$$\tan.\left(\frac{\pi}{4}+\phi'\right)=\frac{\cot.\left\{\frac{\pi}{4}-\left(\phi-\frac{\delta}{2}\right)\right\}}{\cot.\left(\frac{\pi}{4}+\frac{\delta}{2}\right)}.$$

4. If a given semicircle be immersed vertically in a fluid, the image will be a semi-ellipse, and find the ratio of the axes.

5. If the reflecting curve be a logarithmic spiral, luminous point in the pole; shew that the density of light at any point of the caustic will $\propto \dfrac{1}{\text{rad. vector}}$.

6. If a pencil of rays diverge from a given point and fall obliquely on a refracting medium bounded by parallel plane surfaces; then the distance between the foci in the primary and secondary planes are as $\cos.^2\phi : \cos.^2\phi'$, where ϕ and ϕ' are the angles of incidence and reflexion.

7. In the interference of two equal and similar undulations; shew that when one is $(p + \frac{1}{2}) \times$ length of wave behind the other (p being a whole number) they will destroy each other.

SIDNEY SUSSEX COLLEGE, May 1829.

1. Let a thin pencil of rays, converging to a given point, fall nearly perpendicularly upon a convex mirror of given radius: find the focus after reflection; and trace the changes in its position, while the focus of incident rays moves from a point in contact with the back of the reflector to an infinite distance.

2. Let a thin pencil of rays, diverging from a given point, fall obliquely upon a curved reflecting surface: find their intersections after reflection.

3. Find experimentally the refracting power of a given solid substance.

4. Find the focal length of a lens, whose thickness is inconsiderable: and shew which kinds of lenses will make parallel rays converge or diverge.

5. Let two double convex lenses be placed with their axes in the same straight line, the distance between them being greater than the sum of their focal lengths: and let a luminous object be placed between them: shew that the distance between the two images will be a maximum, when the object divides the distance between the lenses into two parts proportional to their focal lengths.

6. When a ray is incident on a single refracting surface, find the longitudinal aberration.

7. Find the difference between the distance of an object when seen by the naked eye, and its apparent distance when seen perpendicularly through a glass window.

8. Find the image of a straight line placed perpendicularly before a lens: and trace its changes when placed at different distances.

9. Explain the unequal refrangibility of light: and find the centre and diameter of the least circle of chromatic dispersion.

10. When an image is formed by reflection in a common looking-glass, explain the aberration of the extreme rays.

11. Construct Gregory's telescope; and find its magnifying power.

12. Explain the formation of the rainbow; find the radius of any given colour in the p^{th} bow: and supposing the rainbow to be formed just at sunset, compare that part of the concave surface of the heavens, which is enclosed within it, with the whole visible hemisphere.

SIDNEY SUSSEX COLLEGE, MAY 1830.

1. Give a brief statement of the two hypotheses which philosophers have made with regard to the propagation of light.

2. Determine the form and direction of a small pencil of rays after being reflected obliquely at a spherical mirror, first, for the primary, next for the secondary plane; and explain clearly the position of these two planes.

3. Find the equation to the caustic when the reflecting curve is an equilateral hyperbola, and parallel rays are incident on the convex surface perpendicular to one of the asymptotes.

4. A given point is placed between two plane mirrors, inclined to each other at a given angle, find the number and position of the images.

5. In the passage of a ray of light through a prism in a plane perpendicular to its axis, show that the deviation is a minimum when the incident and emergent rays make equal angles with the sides of the prism.

6. Find the direction of a ray after refraction through a lens of inconsiderable thickness. What lenses will make parallel rays diverge, and what converge?

7. Determine the analytical relation that must hold between the focal lengths of two lenses in order to form an achromatic combination; why does not the application of this succeed in practice?

8. A short-sighted person can see distinctly at the distance of six inches, find the nature of a glass which will enable him to see distinctly at a distance of eighteen inches.

9. A ray of light is incident on a reflecting surface of revolution at a point x, y, parallel to the axis, show that the distance from the origin of the intersection of the reflected ray with the axis

$$= x - y \cdot \frac{1-p^2}{2p} \left(\text{where } p = \frac{dy}{dx}\right).$$

Apply the case to the paraboloid.

10. Explain the formation of the primary and secondary rainbows, and determine from observations on the rainbow the ratios of refraction, for any coloured light, between air and water.

11. Explain the magic lantern; and find the least distance of the object and the screen at which the image can be formed.

12. Construct Cassegrain's telescope, find the magnifying power, and the greatest field of view, and explain the advantage of this telescope over that of Gregory.

St. PETER'S COLLEGE, May 1831.

1. What is meant by intrinsic, and apparent brightness? How is the former estimated?

2. In direct reflexion at a spherical mirror

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r},$$

u and v being the distances from the surface, of the foci of incident and reflected rays.

3. When rays are incident obliquely on a spherical reflector, explain fully the nature and position of the focal lines in the primary and secondary planes; and shew that for the primary plane

$$\frac{\cos.\phi}{u} + \frac{\cos.\phi}{v} = \frac{2}{r},$$

ϕ being the angle of incidence; and for the secondary plane

$$\frac{1}{u} + \frac{1}{v'} = \frac{2 . \cos.\phi}{r}.$$

4. If in the last question O be the centre of the reflector and the focus of incidence Q be so situated that QO is parallel to the surface of the mirror at the point of incidence, determine accurately the form of a section of the reflected pencil between the two foci, made by a plane perpendicular to the axis of the reflector.

5. When rays are incident directly on two parallel plane refracting surfaces, determine the position of the focus of refracted rays.

6. When rays are incident obliquely on a spherical refractor, prove the formula

$$\mu . \frac{\cos.\phi'}{\cos.\phi} . \frac{\cos.\phi'}{u'} = \frac{\cos.\phi}{u} - \left(\mu \frac{\cos.\phi'}{\cos.\phi} - 1\right)\frac{1}{r},$$

for the primary plane; and

$$\frac{\mu}{u'} = \frac{1}{u} - \left(\mu \frac{\cos.\phi'}{\cos.\phi} - 1\right)\frac{\cos.\phi}{r},$$

for the secondary plane.

7. Explain fully the nature of spherical and chromatic aberration, and how they coexist in the same pencil.

Find the least circle of chromatic aberration in a lens.

In what proportion will the imperfectness of vision through the lens, arising from these aberrations, be remedied by diminishing the aperture?

8. Explain why an object can be seen distinctly through a lens only when placed between the lens and it's principal focus, the distance of the eye from the lens being less than its focal length.

If the distance of the eye from the lens be greater than the focal length, find the limiting positions in which the object may be placed so that it's image may be seen distinctly.

9. If a pencil of converging rays fall upon the first of a combination of two lenses separated by a given interval, in such a manner that they shall emerge parallel from the second lens, determine the focal length of the single lens which in the position of the first of the above lenses shall be equivalent to their combination.

10. Describe the astronomical telescope; and mention the different kinds of eye-pieces commonly used with it, and their advantages and disadvantages.

11. If $\frac{1}{n}$th of the rays incident on each lens of a telescope be transmitted by it, how will the apparent brightness of an object be affected by the use of Huyghens' eye-piece instead of a single lens of the same focal length as the eye-glass of the eye-piece, the focal length of the field glass being twice that of the eye-glass.

12. State concisely the principal postulata of the undulatory theory of light, and whence are derived the strongest reasons for preferring it to the Newtonian theory.

ASTRONOMY,

AND

SPHERICAL TRIGONOMETRY.

TRINITY COLLEGE, 1820.

1. Explain the construction and use of the astronomical quadrant and circle, and state the advantages of the latter instrument over the former.

2. What is the practical mode of drawing a meridian line? During what time of the year will the operation be the least liable to error?

3. By what observations may the figure and dimensions of the Earth be determined?

4. Explain the common modes of determining the latitude of any place on the Earth's surface. Solve the requisite triangles when we have two observations of the Sun's altitude on the same day, and the time elapsed between the observations.

5. What is the difference between a sidereal and solar year? Explain briefly the approximate mode of obtaining the length of each?

6. Assuming the solar year $= 365^d . 5^h . 48^m . 48^s$, what are the necessary corrections of the civil year, in order to make it always nearly coincident with the solar?

7. Assuming the truth of the Copernican system, explain

(1). The phases of Venus,

(2). Her retrogradations,

(3). Her appearance sometimes as a morning and sometimes as an evening star.

8. Find the Sun's altitude at six o'clock on the longest day.

9. Required the time of a given day when the Sun's altitude will be the same at two given places on the same meridian, and on the same side of the equator.

TRINITY COLLEGE, 1820.

1. Explain the construction and use of the vernier. Within what limits may angles be read off by an instrument of which the arc is subdivided to 20′, and 20 divisions of the vernier are equal to 19 of the arc?

2. Explain the mode of correcting a small error in the meridian plane by observations made with a transit instrument on a circum-polar star. Supposing the time between the lower and upper transit $= T$, and between the upper and lower $= T + t$, work out the proper correction.

3. Determine the obliquity of the ecliptic by meridian altitudes taken on successive days before and after the solstice, and apply the proper corrections.

4. Explain the mode of determining the lengths of a sidereal and solar year.

5. Assuming the length of a solar year to be *365d. 5h.* 48′.48″ determine the correction of the eivil year in order that it may always nearly coincide with the solar.

6. Under what circumstances is a star said to rise or set cosmically, achronically, and heliacally?

7. "Hesiod says that sixty days after the winter solstice the star Arcturus rose at sun-set: from which it follows that Hesiod lived about 100 years after the death of Solomon" (Sir I. Newton's Chronology). Exhibit the calculations on which this conclusion is founded.

8. When the Sun is in either of the equinoctial points, determine the locus of the extremities of the shadow of a perpendicular style on a horizontal plane.

9. When the Sun is in either of the equinoctial points and the style of the dial is perpendicular to the horizontal plane on which the hour-lines are drawn; determine the construction of the dial.

10. Upon what experiments does it appear that in the passage of a ray of light through a variable medium like our atmosphere, the sine of incidence is to the sine of refraction in a constant ratio.

11. Explain the mode by which Bradley obtained the following formula:

$$\text{Refraction} = \frac{a}{29{\cdot}6} \times \tan.(z - 3r) \times 57'' \times \frac{400}{350 + 4h}$$

a = altitude of the barometer in inches
$29{\cdot}6$ = mean altitude of do.
z = zenith distance
$r = 57'' \times \tan. z$
h = height of Fahrenheit's thermometer in inches.

12. Define parallax, and determine the law of its variation for the same body at different altitudes.

13. Explain the mode by which (a) the effect of parallax in right ascension may be observed, and prove that the horizontal parallax $= \dfrac{15 \times a \times \cos.\text{dec}^{\text{n}}.}{\cos.\text{lat.} \times \sin.\text{hour-angle}}$.

14. If the velocity of the Earth be in a finite ratio to the velocity of light; (1) find the direction in which a telescope must be held in order that a given heavenly object may appear in its axis: (2) On the same hypothesis shew that a ray of light proceeding from a heavenly body must strike the retina at a different point from what it would do if the eye of the spectator were at rest; and therefore by the laws of vision that the apparent place will differ from the true place of the body. Shew that the quantity and direction of aberration are the same on either of the two preceding considerations.

15. Write down the expressions for the aberration in latitude and longitude, and determine for any given star when the corrections are positive, and when negative.

16. Bradley observed,

(1). "That each star was farthest north when it came to the meridian about six o'clock in the evening, and farthest south when it came about six in the morning."

(2). "That in both stars the apparent difference of declination from the maxima, was always nearly proportional to the versed sine of the Sun's distance from the equinoctial points."

Confirm these observations, and shew that they only apply to stars situated near the solstitial colure.

17. Prove that in elliptical orbits of small eccentricity, the greatest equation to the centre is twice the eccentricity.

18. Explain the mode of correctly determining the longitude of the Earth's apogee: and state at what era in the history of mankind the line of the apsides coincided with the line of the equinoxes.

19. Given the place of a planet as seen from the Earth, find its place as seen from the Sun, exhibiting the formulas of heliocentric latitude and longitude.

20. Account for the moon's vibration in longitude.

21. Find the lunar and solar ecliptic limits: and thence determine the greatest and least number of eclipses, of either kind, that can happen in one year.

22. Suppose the Moon's right ascension to be exactly known, when the Sun is on the meridian; determine when the Moon's centre will be on the meridian.

23. Determine the difference of the longitudes of two places on the Earth's surface, by observations on the passage of the Moon's centre over the meridian.

24. If a small error be made in the assumed distance between two meridians, shew how that error may be corrected by observations on the occultation of a fixed star.

TRINITY COLLEGE, 1821.

1. If two sides and the included angle of any spherical triangle be respectively equal to two sides and the included angle of another; then the third side of the one shall be equal to the third side of the other.

2. The angles at the base of an isosceles spherical triangle are equal.

3. The sum of any two angles of a spherical triangle is greater than the third angle by a quantity which is less than 180°.

4. Prove the truth of Napier's two rules (without taking the fundamental propositions for granted) when the complement of the hypothenuse is the middle part.

5. Assuming the truth of Napier's two rules in any one case, shew, by substitution in the complemental triangle, that they must necessarily be true in both the other cases.

6. In any spherical triangle in which A, B, C are the angles; a, b, c the opposite sides respectively, and $S = \frac{a+b+c}{2}$.

(1). $$\log.\tan.\frac{A}{2} = \frac{1}{2}\{20 + \log.\sin.(S-b) + \log.\sin.(S-c) - \log.\sin.S - \log.\sin.(S-a)\}.$$

(2). $$\tan.\left(\frac{a+b}{2}\right) = \frac{\cos.\left(\frac{A-B}{2}\right)}{\cos.\left(\frac{A+B}{2}\right)} \times \tan.\frac{c}{2}.$$

and

$$\tan.\frac{a-b}{2} = \frac{\sin.\left(\frac{A-B}{2}\right)}{\sin.\left(\frac{A+B}{2}\right)} \times \tan.\frac{c}{2}.$$

(3). Given A, B, c; required C without the determination of a, b.

TRINITY COLLEGE, 1821.

1. Give a short account of the Copernican system, and shew how the principal phenomena presented by the Sun and Moon may be explained by it.

2. In a common mountain barometer, the altitude of the mercury is measured on a scale of inches subdivided into 20 equal parts, and accompanied by a sliding vernier in which 25 equal

parts correspond to 24 of the first scale. Explain the mode of reading off.

3. In the North Polar expedition, when the needle refused to traverse, a sun dial was erected in the place of the compass. Explain the mode in which the steerage of the ship might be conducted by that instrument.

4. Determine the hour of any given day when the Sun passes the horizontal wire of a telescope in the least time possible.—Find the time during which the Sun's disc passes the meridian on the same day. — Determine also the field of view of the telescope, having given the whole time of his appearance in it during the transit.

5. Determine when the Sun's centre is on the meridian by observations of equal altitudes before and after noon. — Apply the proper correction for the change of declination between the observations.

6. Given the latitude of the place; find the time of rising of a known star.

7. Deduce the Sun's variation in declination near the solstice.

8. Find the variation of altitude of the Sun, when very near the meridian, for a small given time.

9. Determine the variation in right ascension and declination:

(1). Having given a small variation in longitude.

(2). Having given a small variation in the obliquity of the ecliptic.

10. Deduce the expression for the area of a spherical triangle, and shew its application in determining the spherical excess in trigonometrical observations on the Earth's surface.

11. In certain latitudes the shadow of a perpendicular style recedes from the meridian for some time after Sun rise. Point out the limits of latitude within which the phenomenon may happen.—Determine also, for a given place, the whole angle through which the shadow recedes on a given day.

12. State the meaning of a sidereal year, a tropical year, and an anomalistic year.—Explain the methods by which the several periods may be correctly determined.

13. Define parallax, and shew its variation for the same body at different altitudes.

14. Determine the parallax of a heavenly body by two observations on the same meridian.

15. Explain the mode of observing the parallax in right ascension.

16. Find an expression for the variation of the Moon's apparent semidiameter at different altitudes.

17. If the velocity of the Earth bear a sensible proportion to the velocity of light, prove that there must be a sensible aberration of the fixed stars.—Determine the variation in the quantity of aberration for the same star.

18. The aberration of a fixed star being observed for a given position of the Earth, from thence deduce the velocity of light.

19. Deduce the expression for the aberration in declination; and confirm the following observation of Bradley, viz.

> "That in both stars (situate near the solstitial colure) the apparent difference of declination from the maxima was always nearly proportional to the versed sine of the Sun's distance from the equinoctial points."

20. Given the mean anomaly of a planet, find the true.

21. Find the time between the conjunctions of two planets. Shew how the calculation may be extended to the time between the conjunction of three planets.

22. Calculate the several dimensions of the umbra and penumbra of the Moon, and find the solar ecliptic limits.

23. Explain the mode of determining the distance between two meridians:

(1). By means of a time-keeper.

(2). By observations on the occultation of a fixed star.

TRINITY COLLEGE, 1822.

1. Ascertain the limit of the sum of the sides of a triangle, and also the limit of the sum of its angles.

2. In two triangles whose sides are a, b, c, and a', b', c' respectively, given that $a = a'$, $b = b'$, $c = c'$, prove the triangles equal in all respects.

3. Explain the formation of the supplemental triangle, and prove its properties;—under what values of its angles does the given triangle coincide with its supplemental triangle, and what ratio does it then bear to the surface of the sphere?

4. In a polygon of n sides whose angles are A, B, C,... P, the surface equals $\{A + B + C + \&c.... P\} - (n - 2)\pi$.

5. In a triangle ABC (right-angled at C) prove, independently of Napier's rule, $\sec.c = \tan.A \,.\, \tan.B$.

6. In any triangle ABC prove

$$2\log.\cos.\frac{C}{2} = 20 + \log.\sin.S + \log.\sin.(S - c) - \log.\sin.a - \log.\sin.b,$$

where $2S = a + b + c$.

7. In any triangle ABC, prove

$$\cos.a = \frac{\cos.A + \cos.B \,.\, \cos.C}{\sin.B \,.\, \sin.C}.$$

8. Give the ambiguous cases in right-angled, quadrantal, and oblique triangles.

TRINITY COLLEGE, 1822.

1. How is it shown that the apparent diurnal motion of the stars is uniformly in circles parallel to one another.

2. Explain the mode of determining the right ascension of a star, the right ascension of none of the others being supposed to be known.

3. The sine of the Sun's declination is a mean proportional between the sines of his altitudes at six o'clock, and when due east.

4. The distance at which the top of a mountain may be seen by means of refraction : the distance at which it could be seen without :: 14 : 13 nearly.

5. If D = apparent zenith distance of a star, r = refraction, and sine of incidence : sine of refraction :: n : 1; then

$$\tan.\frac{r}{2} = \frac{\cot.D - \sqrt{\{\cot.^2 D - (n^2 - 1)\}}}{n + 1}.$$

6. The difference between two contiguous degrees on the Earth's surface is a maximum, when the middle latitude = 45°.

7. If in any latitude λ, the angle which a plumb-line makes with the semi-diameter of the Earth = δ, and a, b, denote the semi-major and semi-minor axes of the Earth, then $\tan.\delta = \dfrac{(a^2 - b^2)\tan.\lambda}{a^2 + b^2 \tan.\lambda}$.

Prove this, and shew that when δ is a maximum, $\tan.\delta = \dfrac{a^2 - b^2}{a^2 + b^2}$ nearly.

8. In the stereographic projection of the sphere, compare (1) the surface contained between two quadrantal arcs perpendicular to the plane of projection: (2) the surface of a zone contained between circles parallel to the plane of projection, with the areas into which they are respectively projected.

9. Shew how the Moon's equatoreal parallax may be determined by two observers on the same meridian.

10. If P = horizontal parallax of a planet, z = its zenith distance, l = its latitude, θ = its angle of position, and ϕ = angle that a vertical makes with the circle of declination passing through it; then

$$\text{parallax in longitude} = \frac{P \sin.z \sin.(\theta + \phi)}{\cos.l},$$

$$\text{and parallax in latitude} = P \sin.z \cos.(\theta + \phi).$$

11. If the obliquity of the ecliptic be assumed = ϕ, and by an observation of the Sun when near the solstice it is found that the difference between his longitude and 90° = l, and the difference between his observed declination at that time and his greatest

declination be assumed $= d$; then d may be determined very accurately from the formula

$$\sin.d = 2 \tan.\phi \, . \, \sin.^2 . \frac{l}{2}.$$

12. How is it shewn that the apparent path of the Sun is an ellipse, of which the Earth is the focus; and that round this the Sun appears to describe equal areas in equal times?

13. If $\frac{CS}{CA} = \sin.\phi$, and the excentric anomaly $= u$, then when the equation of the centre is a maximum, $\cos.u = \frac{(\cos.\phi)^{\frac{1}{2}} - 1}{\sin.\phi}$; and the greatest equation of the centre

$$= 2 \sin.^{-1} \left\{ \frac{\sin.\frac{\phi}{2} . \sin.u}{(\cos.\phi)^{\frac{1}{4}}} \right\} - \sin.\phi \, . \sin.u.$$

14. Trace the changes of the equation of time at different parts of the year.

15. Construct a vertical east or west dial.

16. There must be two solar eclipses in a year, and there may be five. There may not be any lunar eclipse, and there may be three.

17. If P and p be the horizontal parallaxes of the Moon and Sun, and M and S their apparent semi-diameters; then half the angle subtended at the Moon's centre by a section of the lunar umbra at the distance of the Earth $= \frac{P(M-S)}{P-p}$; and by a similar section of the penumbra $= \frac{P(M+S)}{P-p}$.

18. Explain the causes of the stationary appearances, and retrograde motions of the superior and inferior planets.

19. Explain the mode of determining the longitude and latitude at sea.

20. Find the path of a star arising from aberration, when the Earth moves in a curve acted on by a force perpendicular to the axis.

21. Supposing a fixed star to have an annual parallax, compute its effects in longitude, and latitude; and shew that each of these will vanish when the corresponding aberration is a maximum.

22. Explain the means by which Bradley separated nutation from the inequalities of precession, and aberration.

23. In what manner were the distance and periodic time of the Georgium Sidus determined soon after its discovery?

24. If p, q, r denote three heliocentric distances of a planet, α, β, γ half the differences between its heliocentric longitudes at those points, and S the area of the triangle formed by joining the extremities of the distances, then the parameter of the orbit

$$= \frac{2pqr \sin.\alpha . \sin.\beta . \sin.\gamma}{S}.$$

TRINITY COLLEGE, 1823.

1. If in a spherical triangle the angles be denoted by A, B, C, and the sides opposite by a, b, c respectively; what is the value of the ratio $\frac{\sin.A}{\sin.a} = \frac{\sin.B}{\sin.b} = \frac{\sin.C}{\sin.c}$ in a form fit for computation?

2. The cases of a quadrantal spherical triangle may be solved by rules like Napier's.

3. Put $S = \frac{a+b+c}{2}$. Prove that $\tan.\frac{1}{2}A . \tan.\frac{1}{2}B = \frac{\sin.(S-c)}{\sin.S}$, and apply this formula to solve the triangle, when a side, an adjacent angle, and the sum of the other two sides are given.

4. Write down Napier's analogies, and apply them to shew the following particulars:

(1). The difference of two angles is less than 180°.

(2). $\frac{1}{2}(a+b)$ and $\frac{1}{2}(A+B)$ are always of the same affection.

(3). $a-b$ and $A-B$ have always the same sign.

5. Deduce, Prop. 47, Book I. Euc. from the corresponding case of a right-angled spherical triangle.

6. The sides of a spherical triangle are each a quadrant. Prove that the distances (α, β, γ) of a point within it from the angles of the triangle are so connected, that

$$\cos.\alpha^2 + \cos.\beta^2 + \cos.\gamma^2 = 1. \quad \text{Rad.} = 1.$$

7. Given the inclination of the plane of a theodolite to the horizon; required the greatest error that can possibly happen in determining the magnitude of an angle.

8. How is it shewn, that the fixed stars are at distances from the Earth infinitely greater than the Sun and planets, and that the Earth is a sphere of nearly 8000 miles in diameter?

9. What are the definitions of the principal points and planes from which distances are measured in Astronomy? Construct a figure in illustration of your definitions.

10. It is observed in the case of the Sun, that sin.R.A. : tan. decl. is always in the same ratio: shew from this, that his motion is all in one plane.

11. What are the causes of eclipses? why do they not return monthly?

12. Determine the hour of the day by observing the Sun's altitude, the latitude, and his declination. How is the longitude found by a chronometer?

13. Give a popular account of the various astronomical corrections.

14. Mention some of the peculiar phenomena of Jupiter's satellites, and the astronomical discoveries to which they have led.

TRINITY COLLEGE, 1824.

1. SHEW how the difference of seasons, and of days and nights is produced by the motion of the Earth about the Sun.

2. Does the Moon revolve upon her axis? What would be the apparent motions and phenomena of the Earth and the Sun seen by a spectator in the Moon?

3. When is Venus a morning, and when an evening star? Shew how it happens that the time when she is stationary is not when she has moved to her greatest apparent distance from the Sun. Are there any phenomena in a superior planet, which correspond to the station and greatest elongation of an inferior one?

4. What was the nature of the change which made a reformation of the Calendar necessary? What is the alteration produced in the heavens by the precession of the equinoxes?

5. Suppose the precession to be 50″ in a year, explain and verify the following calculation, which Newton makes the foundation of his chronology.

In A. D. 1689, the first star of the constellation Aries had its longitude 25° 35′ of the sign ♈: the last star of the same constellation had its longitude 17° 53′ of the sign ♉. Now the equinoxial colure at the time of the Argonautic expedition passed exactly through the middle of this constellation. Hence it appears, that the Argonautic expedition took place 2645 years before A. D. 1689.

6. The Moon's distance from a fixed star observed at 6 o'clock P. M. was found to be 50°. In the Greenwich tables, the distance 50° of these objects corresponds to 11 o'clock A. M. What is the longitude of the place of observation?

7. Are the apparent places of the fixed stars affected by the motion of the Earth? What is meant by the parallax of the fixed stars? Has it been discovered by observation? Does any argument for the motion of the Earth depend upon its discovery?

8. At a certain place within the Arctic circle, the Sun did not set for two months: what was the latitude?

9. Two stars, α and β, are so situated, that when α rises they are in a vertical line with one another: when α sets they are in a horizontal line with one another. Find the latitude of the place—(If declination of $\alpha = 45°$, latitude $= 30°$).

10. Prove Napier's rules, hypothenuse being middle part.

11. Prove the expression for the sine of the angle of a spherical triangle in terms of the sides.

12. The sides of a spherical triangle are each $= 111° \, 28'$. Find its angles, and shew that its area is $\frac{1}{4}$ the surface of the sphere. ($111° \, 28' = 2 \times 54° \, 44'$, and $\tan. 54° \, 44' = \sqrt{2}$).

13. On the surface of a given sphere, to draw a great circle touching a given circle, and passing through a given point.

14. A hill slopes to the south-east, at an angle of 30°. Shew that the path of the shadow of a fixed object upon it when the Sun is in the equator will be a straight line, and find the position of this line.

TRINITY COLLEGE, May 1831.

1. Explain the Ptolemaic system of the world. Point out its principal defect, and show how it might be made to coincide with that of Tycho Brahé.

2. Give some account of the following lunar inequalities. The evection, the variation, the annual equation, the equation of long period, and the secular acceleration.

3. The length of the longest day at a given place is fourteen hours and a half. Find the latitude, supposing the obliquity of the ecliptic $= 23° 28'$.

$$\begin{aligned} \log.\tan.23°.\,28' &= 9{\cdot}63761, \\ \log.\cos.71\,.\,15 &= 9{\cdot}50710, \\ \log.\tan.36\,.\,31 &= 9{\cdot}86949. \end{aligned}$$

4. The depression of the horizon is observed from the summit of a mountain; find its altitude, taking into account the terrestrial refraction.

5. Find the length of twilight on the day of the summer solstice at latitude 45°.

$$\begin{aligned} \log.\tan.\ 23°.\,28' &= 9{\cdot}63761 = \log.\cos.64°.\,44', \\ \log.\tan.\ 77\,.\,47 &= 0{\cdot}66476, \\ \log.\sin.109\,.\,46 &= 9{\cdot}97363, \\ \log.\sin.\ 1\,.\,46 &= 8{\cdot}48896, \\ \log.\sin.\ 64\,.\,46 &= 9{\cdot}95645, \\ \log.\sin.\ 43\,.\,14 &= 9{\cdot}83567. \end{aligned}$$

6. Find the magnetic variation from observations of the Sun's ortive or occasive amplitude.

7. Show that in finding the time by absolute altitudes the observations should be made near the prime vertical.

8. The length of the sidereal day has not varied one second since the time of Hipparchus. How can this be ascertained?

9. The Moon's right ascension being as follows:

1831. June 16,	0^h	......	$166^o.\ 12'.\ 38''\cdot0$,
	12	......	172 . 35 . 27 ·2,
17,	0	......	178 . 53 . 12 ·4,
	12	......	185 . 6 . 27 ·2,
18,	0	......	191 . 15 . 45 ·9.

Find her A. R. on the 17^{th} at $9^h.\ 47^m.\ 3^s\cdot4$.

10. Explain the method of least squares, and show how it is applied to the correction of astronomical tables.

11. A number of circummeridional altitudes of a planet having been observed, reduce them to the meridian.

12. Given the right ascension and declination of a star, find its longitude and latitude.

13. Explain the construction, uses, and various adjustments of the transit instrument. Show how to place it in the meridian by means of the observation of circumpolar stars.

14. Give some account of the different methods for the determination of the constants in the expression for refraction.

15. Find the parallax in right ascension, and show how in the case of the Moon, the ellipticity of the terrestrial meridians may be taken into account.

16. Given the right ascension and declination of a star for the present year, show how to find the time of year at which it rose heliacally in some preceding century.

17. Find the formulæ for the aberration of the fixed stars in longitude and latitude, and compare them with the respective formulæ for their annual parallax.

18. Explain the phenomena caused by the Earth's atmosphere in lunar eclipses, and calculate the effect produced upon the diameter of the shadow.

19. Show how terrestrial longitudes may be deduced from the observation of occultations of fixed stars by the Moon.

20. The pole star has been observed off the meridian; let the

observed zenith distance $= z$,

hour angle $= t$,

apparent north polar dist. $= p$,

co-latitude of the place $= \phi$.

then $\phi = z + p.\cos.t - \frac{1}{2}p^2.\sin.^2 t.\cot.z.\sin.1''$
$+ \frac{1}{3}p^3.\sin.^2 t.\cos.t.\sin.^2 1''\ldots$

21. Given the apparent distance between the Moon's centre and a star, as well as the apparent altitude of each, to find the true distance.

22. From the passage of a planet through its nodes, to find the longitude of the perihelion and the excentricity of the orbit.

23. Deduce the excentricity of the solar orbit from the greatest equation of the centre.

24. Let $v =$ the anomaly of a comet calculated in a parabolic orbit, the anomaly in an elliptic orbit of great excentricity may be deduced, by adding to v the angle θ, where θ is found from the equation

$$\sin.\theta = \frac{1}{10}(1 - e).\tan.\frac{v}{2}\left(4 - 3\cos.^2\frac{v}{2} - 6.\cos.^4\frac{v}{2}\right),$$

e being the ratio of the excentricity to the semi-axis major in the ellipse.

TRINITY COLLEGE, May 1832.

1. Explain the way in which Ptolemy represented the first inequality of the Sun and planets.

2. Investigate the correction for the Sun's motion in declination to be applied to the instant of noon, concluded from equal altitudes observed on each side of the meridian.

3. Show how to determine the azimuth of a terrestrial signal.

4. Explain the different causes of the equation of time, and give an expression for it.

5. The Sun's declination being 12°. N. the twilight just lasts all night—find the latitude of the place.

6. In north latitude 43°. 30′: the Sun's declination being 15°. 10′. N. the Sun's centre at setting bears by compass W.N.W. find the magnetic variation

$$\log.\sin.15°.\,10' = 9{\cdot}41768, \qquad \log.\cos.43{\cdot}30 = 9{\cdot}86056,$$
$$\log.\sin.21°.\,8' = 9{\cdot}55712.$$

7. Assuming that the refraction (r) for the zenith distance (ζ) may be thus expressed $r = A.\tan.(\zeta - nr)$ deduce the expression $\tan.nr = \tan.nR\tan.\frac{\phi}{2}$ where R is the horizontal refraction, and ϕ is found from the equation $\tan.\phi = \sin.2nR\tan.\zeta$.

8. Explain the method adopted for determining the horizontal parallax of Mars, by observations made nearly under the same meridian.

9. Investigate the formulæ for the annual precession of the fixed stars in right ascension and declination.

10. Show that the effects of nutation may be represented by supposing the apparent pole of the equator to revolve round the mean, in an ellipse, the centre of which is occupied by the latter, and the major axis of which is a tangent to the circle of latitude passing through the poles of the equator and ecliptic, the minor axis being a tangent to the circle on which the mean pole of the equator moves parallel to the ecliptic.

11. The place of a star being taken from a modern catalogue, show how to calculate its place exactly for a very remote time.

12. Show that when a star is in quadratures with the Sun, the aberration in latitude is a maximum, and when in syzygies a minimum.

13. The Sun's meridian altitude having been observed for several days before and after the solstice, show how to reduce the observations to the instant of the solstice.

14. Give an account of the construction, adjustments, and uses of Hadley's sextant.

15. In observing circum-meridian altitudes, show that the reduction to the meridian is of this form,

$$\frac{\sin.\Delta\sin.D}{\sin.\zeta}\cdot\frac{2''.\sin.^2\frac{1}{2}P}{\sin.1''}.(1+2r'),$$

where Δ = declination of the star,

ζ = the zenith distance,

D = the colatitude of the observer,

P = the hour angle,

$$r' = \frac{r}{86400 - r},$$

r being the number of seconds lost in a day by the clock or watch used.

16. In observing altitudes with a circle the plane of which is not exactly vertical, investigate an expression for the error on the altitude, given the deviation from the vertical.

17. Calculate the augmentation of the Moon's apparent semi-diameter depending on her altitude, supposing the parallax in altitude known.

18. Show how to calculate the different phases of a lunar eclipse.

19. Explain the method of finding terrestrial longitudes by the transits of the Moon and moon-culminating stars.

20. Supposing the inclination of a planet's orbit and the longitude of the node known, show how to find, from an observed geocentric longitude and latitude, the radius of the orbit and the distance from the node.

21. From the laws of the planetary motions deduce an expression for the mean in terms of the excentric anomaly.

22. From three successive observed positions of a spot on the Sun's surface determine the inclination of the solar equator to the ecliptic, and the time of rotation.

23. The Sun and Solar system being supposed in motion towards a given fixed star; deduce formulæ for the variations in right ascension and declination of other stars.

24. A comet being supposed to move in a parabola, call t the time between two successive observations, k the chord of the arc comprised between the two places, r, r', the two radii vectores, $\mu = 6\sqrt{(M+m)}$, M being the mass of the Sun, m of the comet, then

$$\mu t = (r + r' + k)^{\frac{3}{2}} - (r + r' - k)^{\frac{3}{2}}$$

St. JOHN'S COLLEGE, 1821.

1. Define a meridian, a meridian line, the zenith, the horizon, and the ecliptic.

2. On a given day, in a given latitude, find the time of day by observing the altitude of a star whose right ascension and declination are known.

3. Explain the method of constructing tables of refraction for any latitude. How may the horizontal refraction be determined?

4. In a given latitude, a circumpolar star whose declination is known passes a vertical circle and repasses it after an interval of twelve hours, mean solar time; find the meridian of the place?

5. Shew how the periodic time of a superior planet may be found. Explain how the periodic time of the Georgium Sidus was determined?

6. In an elliptic orbit of very small excentricity, prove that the horizontal parallax at the mean distance will be an arithmetic mean between the greatest and least horizontal parallax.

7. Find the parallax of a heavenly body by observations made out of the plane of the meridian.

8. Determine from the requisite data the duration of a lunar eclipse. Point out what artifice in the analytical solution corresponds to the introduction of the relative orbit in the geometrical. Find the limits to the whole number of eclipses that may happen in a year.

9. Explain the principle of the repeating circle, and shew how it may be applied to find the meridian altitude of the Sun with accuracy.

10. Construct an horizontal dial.—In lat. 60°. the shadow of the style arrives at the ten o'clock hour-line 2″ sooner, and at the two o'clock hour-line 2″ later than it ought. Required in seconds of a degree the dip of the dial-plate from the horizontal plane?

11. From what arguments is it inferred that the Earth is round, and from what, that it is not spherical? Its true form being supposed an oblate spheroid, and its dimensions known, find in what latitude a degree of longitude and latitude are equal.

12. At a given place, it was observed that the interval which elapsed between the rising of a star, and its crossing the prime vertical, was half that between rising and crossing the meridian. Having given the hour of the star's rising, and the day of the year, find the declination and right ascension of the star.

St. JOHN'S COLLEGE, May 29, 1822.

1. On what day of the year in latitude (l) does the length of the day equal the length of the longest day in latitude 45°?

2. Supposing the latitude of Greenwich Observatory to be exactly determined, and the zenith distances of some star near the zeniths of Greenwich and Cambridge to be likewise determined, find the latitude of the Observatory at Cambridge. Why is this method more accurate for stars near the zenith than for those which are not so?

3. A horizontal dial is constructed for a given latitude; in what latitude would it serve for a vertical south dial?

4. At a given place the Sun and a Star are found at nine o'clock to have the same azimuth which is observed; the interval between their transits over the meridian is also noted; find the day of the year, and the altitudes of the Sun and Star, when they cross the meridian.

5. Explain the construction and use of the zenith sector, and how it is applied to determine the error of collimation in mural quadrants.

6. Supposing no division of an instrument to coincide with a division on its vernier, shew how the defect is remedied by a micrometer screw.

7. Determine the right ascension of a heavenly body by observations on the Sun when near the vernal and autumnal equinoxes (Flamstead's method), and state the advantages of this method.

8. Shew how the mean daily rate of the clock is to be determined; and in the last question supposing t the apparent difference in right ascension of the Sun and Star on a day when $+ r$ is the

daily rate of the clock, and t' the apparent difference on the day of the second observation when $-r'$ is the daily rate; shew how t and t' are to be corrected.

9. Explain the method of computing parallax by observations made by two observers in given latitudes on the same meridian. What correction must be applied, if the observers are on different meridians?

10. Prove that the annual precession in right ascension is $50''.1$ (cos.I + sin.I. sin. right ascension . cotan. north polar distance) (I being the obliquity). Would n times the above expression be the precession for n years, n being considerable? How might the precession for any considerable period be accurately computed?

11. Prove that $\cot.P = \cos.l.\cos.I.\sec.L - \sin.l.\tan.L.$ P being the angle of position, I the obliquity of the ecliptic, L and l the longitude and latitude of a star; and supposing a small error in determining I, find the error of P.

12. State the elements of a planet's orbit, and shew how the longitude of the aphelion, and the time at which the planet is in the aphelion may be determined.

13. Investigate the following formula for computing the time of the Moon's transit over the meridian:

$$\text{Time of transit} = t + \frac{A-a}{24}.t + \left(\frac{A-a}{24}\right)^2.t^2 + \&c.$$

t being the difference of right ascension of Sun and Moon on the preceding day at noon, a and A their diurnal increments of right ascension.

14. Prove that the quantity of aberration $= c \times$ sine of the Earth's way (c being some constant quantity), and shew how Bradley determined the value of c.

15. If p and e be the polar and equatorial diameters of the Earth, E and L the lengths of a degree of the equator and of a curve perpendicular to the meridian in latitude (l),

$$\frac{p}{e} = 1 - \frac{L-E}{E\sin.^2 l}$$

the ellipticity being supposed small.

1. The sum of the sides of a polygon described on the surface of a sphere, the sides being arcs of great circles, is less than the circumference of a great circle.

2. Solve that case of quadrantal triangles in which the complement of one of the sides is the middle part.

3. If A' be the angle contained between the chords of an isosceles spherical triangle, and A the spherical angle, then

$\sin.\frac{A'}{2} : \sin.\frac{A}{2} :: \cos.\frac{b}{2} :$ radius. b being one of the equal sides.

4. Having given two angles and the included side, find the third angle in a form adapted to logarithmic computation, without first finding the other sides; and apply your formulæ to find the third angle in a triangle, two of whose angles are 131° 30′, and 51° 30′ 16″, and the included side 80° 16′.

5. Prove that the orthographic projection of a circle on a plane inclined to its own plane is an ellipse. What must be the inclination of the plane that the area of the ellipse may be half that of the circle.

6. The diameter of any small circle of the sphere, perpendicular to the primitive, projected stereographically, is equal to twice the tangent of its distance from the nearest pole.

St. JOHN'S COLLEGE, 1824.

1. In a spherical triangle express the cosine of an angle in terms of the sines and cosines of the sides.

2. If the sides of a spherical triangle be a, b, c, and (d) be the length of an arc which bisects the angle (C) and is terminated in the opposite side (c),

then $\cos.\frac{C}{2} = \frac{\sin.(a+b)\tan.d}{2\sin.a \sin.b}$. Required a demonstration.

3. In the stereographic projection of a great circle, prove that the tangent of any arc terminated at the plane of projection, is projected into a straight line of equal length.

4. If at noon the Sun is elevated 45°. 58′ above the horizon, and at midnight depressed 119°. 38′ below it; what is its declination, and the latitude of the place?

5. Given the Sun's azimuth, and the hours of the day, at a place whose latitude is known, investigate formulæ by which his altitude may be determined.

6. Shew that on the day of shortest twilight the Sun's azimuths at the end and beginning of the crepusculum are supplements of each other.

7. Having given the difference of altitude of two known stars when upon the same vertical whose azimuth is also given: determine from thence the latitude of the place.

8. Deduce expressions for the aberration of a given star, in north polar distance and right ascension, on any given day.

9. In any proposed latitude, having given the angular distance of two hour circles from the meridian, it is required to find what must be the declination of that star whose variation in altitude during its passage from one hour circle to another shall be the greatest possible.

10. Explain from whence the equation of time arises.

11. State clearly the origin and extent of lunar librations.

St. John's College, 1825.

1. If A, B, C, be the angles, a, b, c, the opposite side of a spherical triangle, b, c, and C being given; determine A by a mode adapted to logarithmic computation.

2. If a and b are nearly equal, and $\alpha = \frac{1}{2}(a + b)$,

$$a = \alpha + \tan.\alpha \tan.\tfrac{1}{2}(A - B) \cot.\tfrac{1}{2}(A + B) \text{ very nearly.}$$

3. Find the radius and place of the centre of a given circle stereographically projected.

4. In a given latitude, find the angle which the diameter of the Earth makes with the plane of the equator.

5. Two known circumpolar stars are observed to attain their greatest azimuths at the same instant on a given day; hence find the latitude of the place and the time of observation.

6. Find when a given star rises heliacally.

7. If the refraction $= a \tan. z + b \tan.^3 z$, z being the apparent zenith distance, determine the coefficients a and b from observation.

8. Explain clearly the cause of the error of aberration, and find the coefficients of aberration arising from the Earth's motion in its orbit, and from the rotation round its axis.

9. From what observations does it appear that the Earth's orbit is an ellipse?

10. Find the inclination of the Sun's axis to the ecliptic, from observations made on the same spot of its disk.

11. Find the length of a tropical, a sidereal, and an anomalistic year, supposing the Sun's mean daily motion in right ascension to be $59'.50''$, and $11''$ the respective annual motion of the equinox and apogee.

12. The heliocentric latitudes of a planet at the time of its transit over the meridian, on four successive days were $3°.15'$, $3°.20'$, $3°.18'$, $3°.9'$; hence find the inclination of its orbit, supposing the intervals between the times of transit equal.

13. The true anomaly (v) may be determined by the area (nt) when the eccentricity is small, by the equations

$$\tan.(u' - \tfrac{1}{2}nt) = \frac{1 - e}{1 + e}, \tan.\tfrac{1}{2}nt$$

$$u = \frac{nt + eu' \sin.u' - e \sin.u'}{1 + e.\cos.u'}$$

$$\tan.\frac{v}{2} = \sqrt{\left(\frac{1 - e}{1 + e}\right)}.\tan.\frac{u}{2}$$

14. State the limits of the number of eclipses that can happen in the course of a year.

St. JOHN'S COLLEGE, May 1831.

1. Given the three sides of a spherical triangle; find the sine and cosine of one of its angles.

2. Given two sides and the included angle of a spherical triangle; find the two remaining angles, and the third side independently, in formulæ convenient for logarithmic computation.

3. If one angle C, and the side opposite c, of a spherical triangle, remain constant, shew that the corresponding variations of the other sides are connected by the equation

$$\cos.A \,.\, \delta b + \cos.B \,.\, \delta a = 0.$$

4. Find the latitude and hour-angle from two altitudes of the Sun, and the time between.

5. Given the latitude and longitude of a star, and the obliquity of the ecliptic; find the angle of position of the star.

6. The orthographic projection of a circle is an ellipse; find its semi-axes.

7. Construct a vertical dial inclined at a given angle to the meridian.

8. Determine the coefficient of refraction by means of solstitial zenith distances of the Sun.

9. Find the azimuth of a terrestrial object.

10. Find the effects of parallax on the latitude and longitude of a known body.

11. Find the effect of aberration on a planet. Compute roughly the Moon's aberration in longitude. Find the coefficient of aberration from observations of a star in the solstitial colure.

12. If the angle of position of a star $= 90^\circ$ or 270°, the precession does not alter its right ascension.

13. Find the time between two successive transits of a planet.

14. Find the geocentric latitude of a planet at conjunction.

15. Given the heliocentric to find the geocentric place of a planet at any time.

16. Find the lunar ecliptic limit.

17. Given the mean anomaly, to find the true, in orbits of small excentricity.

St. JOHN'S COLLEGE, May 1831.

1. The sine of the arc, which is drawn from the right angle of a spherical triangle perpendicular to the hypothenuse, is a mean proportional between the tangents of the segments of the hypothenuse.

2. If (a) be one of the (n) sides of a regular spherical polygon, its surface (S) may be found from the equation

$$\cos.\frac{1}{n}\left(\pi - \frac{S}{2}\right) = \cos.\frac{\pi}{n}.\sec.\frac{a}{2}$$

3. A ship's longitude is $112^\circ. 45'$ east of Greenwich, and the apparent time is $5^h. 16^m$; find the apparent time at Greenwich.

4. The latitude (λ) and longitude (l) of a star are such, that

$$\tan.\lambda = 2\tan.\omega.\sin.l;$$

determine the orthogonal projection on the ecliptic of the locus of all such stars.

5. Given the difference of the greatest and least azimuths of the Sun at rising, determine the latitude of the place: and shew, that for all places on the equator this difference equals twice the obliquity.

6. Explain how the deviation of the plumb-line from the vertical may be determined by observations on a star, made at the north and south points of the base of a mountain.

7. If $\Delta =$ north polar distance of the polar star, and (a) its altitude at an hour angle (h), the latitude of the place of observation

$$= a - \Delta\cos.h + \tfrac{1}{2}\Delta^2\sin.^2 h\tan.a.$$

8. Investigate Littrow's method of finding the refraction.

9. Explain Hadley's Sextant. By what alteration in the position of the horizon-glass may an angular distance greater than 120° be observed?

10. The equations to the locus of all stars, whose aberration in right ascension and declination $= 0$, when the Earth's longitude is α, are

$$x^2 + y^2 + z^2 = 1,$$

$$zy - x = (zx + y)\tan.\alpha,$$

the stars are referred to the surface of a sphere (radius = 1), the plane of xy lies in the ecliptic, and the line of equinoxes is assumed as the axis of x.

11. Investigate an approximate formula for determining the longitude of a place by observations on the Moon's transit over the meridian. Why in determining the longitude at sea would the results, obtained by clearing the Sun's distance from a fixed star, be less accurate than those obtained from the common method?

12. An accurately graduated horizontal dial is fixed with a small error (α) in azimuth; find the error for any given hour-line; and shew that at 3 o'clock, the error $= \dfrac{\alpha}{1 + 2\tan.^2 l}$ nearly, where l = the latitude of the place.

13. Given the declinations of a planet at 12 o'clock on the 19th, 24th, 29th of May, and the 3rd, and 8th of June = 48′, 1° 25′, 2° 12′, 3° 8′, 4° 12′ respectively; find the declination at 12 o'clock on the 31st of May.

St. JOHN'S COLLEGE, May 1832.

1. Prove that any two sides of a spherical triangle taken together are greater than the third.

2. State and prove Napier's Rules for the solution of right-angled spherical triangles, when one side is the middle part.

3. Given two angles and the included side of a spherical triangle; find the remaining sides and angle, in forms adapted to logarithmic computation.

4. When the vertical plane in which a transit instrument moves, nearly coincides with the meridian, find the deviation.

5. If a, l, be respectively the right ascension and longitude of the Sun, and ω the obliquity of the ecliptic, investigate the following series for the reduction of the ecliptic to the equator:

$$l - a = \tan.^2\frac{\omega}{2} \cdot \sin.2l - \tfrac{1}{2}\tan.^4\frac{\omega}{2}\sin.4l + \&c.$$

6. Find the time from an observed altitude of the Sun; and having given a small error in the observed altitude, find the corresponding error in the computed time.

7. Find the length of an arc of the meridian on Mercator's projection, supposing the earth spherical.

8. Construct a vertical south dial.

9. Explain the effect of refraction on the place of a heavenly body; and shew how the refraction may be corrected for any change in the thermometer and barometer.

10. Find the parallax of a heavenly body by observations made in the plane of the meridian.

11. Find the precession of a given star in right ascension and declination.

12. Shew that the curve which the Earth describes round the Sun is an ellipse; and find the relation between the true and mean anomalies.

13. Find a planet's distance from the Sun, and the argument of latitude at the time of conjunction.

14. Explain the Moon's librations; and the phenomenon of the harvest moon.

15. Find the time, magnitude, and duration of a lunar eclipse.

ST. JOHN'S COLLEGE, MAY 1832.

1. X, Y, Z are any three points on the surface of a sphere; P, Q, R any three points in a great circle of the sphere: prove that

$$\frac{\cos.PX.\cos.QY-\cos.PY.\cos.QX}{\cos.PY.\cos.QZ-\cos.PZ.\cos.QY}=\frac{\cos.RY.\cos.QX-\cos.RX.\cos.QY}{\cos.RZ.\cos.QY-\cos.RY.\cos.QZ}$$

2. QR is the stereographic projection of a great circle, P the projection of its pole; PQq, PRr straight lines meeting the primitive in q, r: the arc projected into QR is measured by qr.

3. Find the equation to Mercator's projection of the ecliptic, the projection of the equator being the axis of x, and ♈ the origin of co-ordinates.

4. On a given day, at a given place, the Sun's light is reflected to the south point of the horizon: find the relation between the N.P.D. and hour angle of a normal to the mirror at any time.

5. The Moon shines on a sun-dial: having given the hour line on which the shadow of the style rests, and the time when the Moon crosses the meridian; find the solar time.

6. Deduce the coefficient of refraction from the greatest and least zenith distances of the same circumpolar star, observed in places having different latitudes.

7. Find the Sun's aberration in longitude when seen by an observer in the Moon.

8. Having given the north polar distance of a planet, and the daily variation of its north polar distance; find the planet's hour angle when its zenith distance is a minimum.

9. Having given the time between the superior and inferior transits of the pole star when viewed directly, and the time between its superior and inferior transits when observed by reflection from the surface of mercury: find the angle between the great circle described by the optic axis of the transit telescope and the meridian, and the zenith distance of the point in which it cuts the meridian.

10. S is the focus, C the centre, and A the apse of an ellipse; AQ a circle having C for its centre, QPM perpendicular to AC, QTR perpendicular to CQ meeting AR the involute of AQ in R, ST parallel to CQ. The time of describing AP, round a centre of force (μ) at S, is equal to $\sqrt{\left(\frac{AC}{\mu}\right)}\,TR$.

11. S is the Sun, E a planet describing a circle, P a comet describing a parabola (vertex A) in the same plane. The comet will appear stationary to an observer at E, when

$$\left(\frac{SA}{SE}\right)^{\frac{3}{2}} \cos.SEP = \sqrt{2}\,.\,(\cos.\tfrac{1}{2}PSA)^2 \left[\sin.SEP \cos.\tfrac{1}{2}PSA - \sqrt{\left\{\left(\frac{SA}{SE}\right)^2 - (\sin.SEP)^2 (\cos.\tfrac{1}{2}PSA)^4\right\}}\right].$$

12. Construct for the places on the earth's surface where a given solar eclipse is visible.

13. The zenith distance of a comet on a given day at

$2^h. 15^m$	is	$37°. 12'$
2 . 49		41 . 10
3 . 47		45 . 13

find its zenith distance at $2^h. 31^m$.

CAIUS COLLEGE, MAY 1830.

1. DEFINE—'rational horizon', 'nonagesimal degree'—'solstitial colure', 'prime vertical' and '*zodiac*'.

2. Describe the transit instrument, and the mode of adjusting it; and shew how to determine the deviation when its plane nearly coincides with the meridian.

3. Determine the refraction at any zen. dist. (z) in a homogeneous atmosphere, and reduce it to the form $r = A \tan.(z - nr)$.

4. Find the parallax in right ascension, and adapt the resulting formula to logarithmic calculation by means of an auxiliary angle.

5. Calculate the effect of precession on the right ascension and declination of a star; in the latter case the effects are contrary on opposite sides of the solstitial colure.

6. Find in general the equation to the aberratic curve, and shew that if the centripetal force on the Earth $\propto \frac{1}{r^n}$, the curvature of the aberratic curve $\propto r^{n-2}$.

7. If v = Earth's longitude at the time t after its passage through apogee, and L = longitude of apogee, demonstrate the system of equations,

$$\tan. \frac{u}{2} = \sqrt{\frac{1-e}{1+e}}, \times \tan. \frac{v-L}{},$$

$$t = \frac{T}{2\pi}(u - e \sin. u); \text{ where } T = \text{periodic time.}$$

8. Prove that the equation of time vanishes four times a year.

9. Determine the time of the year when a given star rises heliacally.

10. If D = perihelion distance of a Comet, v = anomaly in time (t) reckoned from the passage through perihelion, then

$$t = \frac{T.D^{\frac{3}{2}}}{\pi\sqrt{2}}\left\{\tan.\frac{v}{2} + \frac{1}{3}\tan.^3\frac{v}{2}\right\},$$

where T = sidereal period of the Earth.

11. Prove that the length of the Moon's shadow in a solar eclipse $= \frac{P-p}{P} \cdot \frac{m}{\sin.(r-mp)}$.

P, p being the horizontal parallaxes of the Moon and Sun, $r =$ apparent semi-diameter of the Sun, $m =$ the Moon's radius, that of the Earth being unity.

12. Determine the phases of an opaque heavenly body illuminated by the Sun's rays.

13. Shew how to determine the Sun's parallax by observations on the transit of Venus.

14. Find the stereographic projections of a plane section of the Earth's surface considered first as a sphere, second as a spheroid, the eye being at the pole.

15. State the principal phenomena from which the Earth's annual motion is most strongly inferred.

PEMBROKE COLLEGE, June 1832.

1. Express the sine and cosine of an angle of a spherical triangle in term of the sides.

2. In any spherical triangle

$\tan.\frac{1}{2}(A+B) = \frac{\cos.\frac{1}{2}.(a-b)}{\cos.\frac{1}{2}(a+b)} \cot.\frac{C}{2}$: also having given a, b, C, find c immediately in a form adapted to logarithmic computation.

3. If θ be an arc of a great circle bisecting angle C of a spherical triangle and terminated by the base, then will

$\tan.\theta = \frac{2\sin.a.\sin.b}{\sin.(a+b)} \cos.\frac{C}{2}$: prove this, and from it deduce the corresponding expression in a plane triangle.

4. When the Greenwich time is $3^{hrs}. 33'. 47''$. P.M., the time at another place is $11^{hrs}. 2'. 45''$. A.M.; find its longitude.

5. If θ be the maximum difference between the true latitude of any place on the Earth's surface, and the latitude of the same place referred to the centre, then will $\tan.\theta = \frac{a^2-b^2}{2ab}$.

6. State the different methods of finding the compression of the Earth. In what way is an arc of the meridian measured, and what latitudes should be selected for the purpose.

7. Find the latitude of any place from observing the times of rising of two known stars.

8. Explain the following phenomena:
The different lengths of day and night.
The change of seasons, writing them down in order of length.
The Harvest Moon and the Moon's phases.

9. Find in what latitude the ratio of the lengths of the shadows of the same vertical rod, at noon on the longest and shortest days, is a maximum.

10. Find the duration of twilight on a given day at a given place: find also the time of year when it is shortest.

11. Explain the calendar having given the length of the tropical year $= 365^{d}. 5^{hrs}. 48'. 51''\cdot 6$: on what day of the week will the first day of the year 2117 be?

12. If m be the maximum lunar nutation of a given star in right ascension, l the corresponding longitude of the Moon's ascending node, then when the longitude of the node equals l' the nutation will be $m \cos.(l' - l)$.

13. Given $m = u - e \sin. u$, the relation between the mean and eccentric anomalies, and $\tan.\frac{\theta}{2} = \sqrt{\left(\frac{1+e}{1-e}\right)}\cdot\tan.\frac{u}{2}$ the relation between the true and eccentric, find θ in terms of m as far as e^3.

14. Enumerate the different methods of finding the longitude, and find it by the transit of the Moon over the meridian.

15. The orbit in which the Earth moves is a parabola, the force acting in lines parallel to the axis, find the curve of a fixed star's aberration.

16. What is the form of the three principal equations of the Moon's motion? In finding u in terms of θ to the second order in the lunar theory, state the form of those terms of the third order that must be considered.

17. The Sun's declination, Sept. 19, at noon was ... 44′.36″ N.
.................................20, ... 20′.56″ N
.................................21, ... 2′.45″ S
.................................22, ... 26 .27″ S.

Find when the Sun was in the equator.

18. Enumerate the arguments for the revolution, rotation and rotundity of the Earth.

19. Describe the nature and use of the Vernier and micrometer screw.

SIDNEY SUSSEX COLLEGE, May 1829.

1. Suppose a meridian line to be drawn on a horizontal plane, by bisecting the equal shadows cast upon it, before and after noon, by a vertical gnomon: shew that from the longest to the shortest day this line deviates from the true meridian line, towards north-west and south-east; and in the other half year, towards north-east and south-west; and find the deviation.

2. Find the latitude of the place, having given the meridian altitudes of a known circumpolar star:

(1). When the star's parallel of declination crosses the prime vertical;

(2). When it does not cross it.

3. On a given day let two altitudes of the Sun and the elapsed time be observed: find both the latitudes which will satisfy these data; and shew by what additional data the ambiguity may be removed.

4. Describe on the concave surface of the heavens the apparent path of the nonagesimal degree of the ecliptic above the horizon of a given place, during one revolution of the Earth: find its limits of altitude, and the corresponding azimuths; its limits of azimuth, and the corresponding altitudes: find also the time which it occupies in passing from one limit of each to the other.

5. On a given day, at a given place, find the mean time of the rising and setting of a known star.

6. At a given place between the tropics, in north latitude, find the time on the morning of the longest day, when the Sun is directly over that point of the horizon, on which he rose.

7. In a given north latitude, on a given day, find the hour when the Sun's azimuth from the south is equal to the hour-angle from apparent noon.

8. Find the effect of the lunar nutation on the right ascension of a given star.

9. Explain the aberration of the fixed stars; and find the aberratic curve, supposing the Earth to oscillate in a cycloid.

10. Find the greatest equation of the centre, in an elliptic orbit.

11. Find when that part of the equation of time, which arises from the obliquity of the ecliptic, is nothing, and when a maximum; when it is additive, and when subtractive.

12. Given three geocentric observations of a comet, find its heliocentric and geocentric distances.

13. Explain fully the changes which take place in the phases of the Moon.

14. Find the lunar and solar ecliptic limits; and the greatest and least number of eclipses of each kind, which can happen in a year.

15. Find the times of the beginning and ending of an occultation of a given star by the Moon:

(1). As seen from the centre of the Earth;

(2). As seen from a given place on its surface.

16. Given the apparent distance between the centres of the Sun and Moon, and their observed altitudes: find the true distance; and hence find the longitude of the place of observation.

17. Explain the stereographic projection of the sphere: and find the centre and radius of the projection of a given great circle.

18. Construct a vertical south-west dial, for a given latitude: and find how long, on a given day, the Sun can shine upon it.

19. Find the time of equinox, and the latitude of the place, from the following data:

1829. March.	Sun's Longitude.	Sun's Meridian Altitude
19th at Noon ...	$11^s . 28^\circ . 39' . 10''$	$37^\circ . 59' . 10''$
20	11 . 29 . 38 . 40	38 . 22 . 51
21	0 . 0 . 38 . 8	38 . 46 . 31
22	0 . 1 . 37 . 34	39 . 10 . 10

SIDNEY SUSSEX COLLEGE, May 1830.

1. State concisely the arguments in favour of the diurnal and annual motions of the Earth.

2. State what is meant by the Earth's compression; and find it from any two measured arcs of the meridian.

3. Having given the times of transit of two known stars, find the quantity and direction of the error of the transit instrument from the plane of the meridian.

4. Explain the use of the Vernier and Micrometer.

5. When the latitude is found at sea from two altitudes of the Sun and the time between, correct for the ship's change of place between the observations.

6. Given the lengths of the shadows, cast on the horizontal plane by a vertical tower at noon, on the longest and shortest days: find the latitude of the place.

7. Any two of the four quantities,—obliquity of the ecliptic, Sun's longitude, right ascension, or declination,—being given, investigate equations for determining the other two: and shew which of the cases will admit of two answers.

8. Find the mean time of the transit of a planet whose motion is retrograde.

9. The latitude of the place being known:

(1). Find the days on which the Sun rises with a given azimuth.

(2). Find the limits within which the azimuth must be given.

10. At a given place, on a given day, at a given hour, let the great circle which joins two known stars be produced to cut the horizon: find the points and angles of intersection.

11. Find the change in the refraction at a given altitude, depending on the barometer and thermometer.

12. The mean right ascension of the star α Aquilæ for the year 1830 is $19^h. 42^m. 29^s \cdot 34$, and its north polar distance $81°. 34'. 26'' \cdot 7$. Find whether its meridian altitude, at a given place, will be greater or less this day next year than it is to-day: shew how the magnitude of the variation may be computed.

13. On a given day find how much, and which way, the right ascension of a known star is affected by aberration; and on what days the effect is nothing or a maximum.

14. Given the mean anomaly of a planet; find its true anomaly.

15. The orbit of a planet being supposed a circle, whose radius is known nearly; approximate to the true radius.

16. Given the synodic time of a superior planet; find its periodic time.

17. A planet is a morning star, and stationary: when it begin to move again, find which way it will move:

(1). Supposing it an inferior;

(2). Supposing it a superior planet.

18. Find the eccentricity of the lunar orbit.

19. In the year 1823 there were seven eclipses, and the first was an eclipse of the Sun: state the others in the order in which they happened, and as near as you can the times of the year at which they happened.

20. Given the periodic times of the Earth and Venus, equal to 365·256 and 224·7 days, respectively, find the periods at which transits of Venus may be expected to recur.

21. Construct a horizontal dial for a place on the equator.

SIDNEY SUSSEX COLLEGE, May 1831.

1. Define the latitude of a place on the Earth's surface: by what angle within the Earth is it measured when the Earth is a sphere, and by what angle when the Earth is a spheroid? Find the latitude from two equal altitudes of the Sun observed before and after noon.

2. Supposing the Earth an oblate spheroid, find its radius, at any point, in terms of the latitude.

3. Investigate Flamstead's method of finding the right ascension of the Sun.

4. Investigate the reduction of the ecliptic to the equator in terms of the longitude.

5. Find the altitude and azimuth of a known star when that arc of the vertical circle passing through it which is intercepted between the star and the ecliptic, is a minimum.

6. Explain the differences between sidereal time, true solar time, and mean solar time: find the length of a sidereal day in mean solar time, and that of a mean solar day in sidereal time.

7. At a given place on a given day, find the time and place of sun-rise, the length of the day and night, the Sun's meridian altitude and midnight depression.

8. On a given day at a given place, find the spherical area swept out by the arc ZS from sun-rise to sun-set.

9. On a given day at a given place, the true time is to be found from observing the Sun's altitude: find at what time in the morning the observation must be made, so that a small error in the observed altitude may have the least effect on the result.

10. On a given day at a given hour, let a globe of given radius be suspended in the air: having given the dimensions of its shadow cast by the Sun on the ground, find the latitude of the place, and the direction of the meridian line. Determine under what circumstances this problem admits of two answers.

11. Explain the construction and use of the transit instrument, and shew how to adjust its axis and line of collimation.

12. Investigate Brinkley's formula for the refraction, the atmosphere being homogeneous.

13. Explain the nature of parallax, and its effects on the hour angle and declination of a known body.

14. Find the aberration of a given star in declination: and trace the changes, arising from this cause, in the meridian altitude of the star, throughout the year.

15. Explain the terms 'Mean Anomaly' and 'True Anomaly:' and having given the mean anomaly, find the true.

16. Shew when that part of the equation of time which arises from the obliquity is additive or subtractive.

17. The orbits of the Earth and planets being circles, shew when an inferior or superior planet will appear stationary, when to be moving direct, and when retrograde.

18. Explain the Moon's librations in longitude and latitude, and account for them.

19. Compute the time of a solar eclipse at a given place.

20. Explain fully the process of finding the longitude by observing the distance between the Moon and the Sun or a fixed star.

21. Explain the stereographic projection of the sphere; and shew that the angle between any two circles on the surface of the sphere is the same as that between their stereographic projections.

22. An horizontal dial constructed for latitude l is fixed in a place whose latitude is $l+\theta$, θ being small when compared with l: find the correction, which must be applied to the time shewn by the dial, in order to obtain the true apparent time.

NEWTON,

AND

CONIC SECTIONS.

TRINITY COLLEGE, 1821.

1. Explain the method of indivisibles, and compare the area of a parabola with that of the circumscribing parallelogram, by that method.

2. Prove that the ultimate ratio of the chord, arc, and tangent, is a ratio of equality.

3. Shew that the curvature of the semi-cubical parabola is infinitely greater, and that of the cubical parabola infinitely less, than that of the common parabola.

4. Compare the forces to two points within a circle, the periodic times being supposed different.

5. Find the variation of the force when the body moves in an ellipse, the force acting in a direction parallel to the ordinates.

6. Find the horizontal velocity in a cycloid, when the force acts parallel to the axis.

7. When is the paracentric velocity a maximum? In what point of all conic sections is it so? Does it admit of a maximum in a circle, the centre of force being in the circumference?

8. Shew fully that if a body move in a logarithmic spiral, the force $\propto \frac{1}{D^3}$; and compare the time of describing this spiral with that of describing a circle at distance SP.

9. Investigate the relation between the centripetal and centrifugal forces, the equation to the curve in which they are equal, and the law of force by which it will be described.

10. Shew that round different centres, the periodic times in all ellipses $\propto \frac{1}{\sqrt{(\text{abs. force})}}$.

11. Required the law of force in a parabola.

12. Determine that point in an ellipse, force in focus, in which the velocity is an arithmetic mean, and also the point in which it is a geometric mean, between the velocities at the greatest and least distances.

13. Force of gravity $\propto \frac{1}{D^2}$, given the periodic time of the Moon, and her distance from the Earth; required how far a body falls in 1″ at the Earth's surface.

14. The rectangle contained by two perpendiculars drawn from the foci of an ellipse to the tangent at any point, is equal to the square of the semi-axis minor.

15. To draw a tangent to any conic section from a given point without, which is not the centre of the hyperbola.

16. In an ellipse whose eccentricity is small, the increment of the radius vector, in moving from the extremity of the axis minor to that of the axis major, varies as the square of the sine of the angle through which it has passed.

17. To find the curve that cuts any number of similar concentric ellipses at right angles.

TRINITY COLLEGE, 1821.

1. State the reasonings by which the following propositions are established:

(1). That the planets with their satellites gravitate to the Sun; the satellites to their planets, and the Moon to the Earth.

(2). That the force acting on the same body in different parts of its orbit, and on different bodies in different orbits, round the same centre of force, varies inversely as the square of the distance from that centre.

2. Prove from the equations $X + \frac{d^2x}{dt^2} = 0$, and $Y + \frac{d^2y}{dt^2} = 0$, that equal areas can only be described in equal times round a point, when the forces acting on the body tend to that point.

3. If the force vary as $\frac{1}{(\text{Dist.})^{\frac{5}{3}}}$, find the time of falling into the centre.

4. If a chain of great length be suspended at the top, its lower end touching the earth, and then be let fall, find the velocity of the chain.

5. Find the velocity and time of a body's descending in an evanescent ellipse towards the centre of force placed in the focus; and supposing two bodies to descend, one in an evanescent ellipse, and the other in a right line, how will each move after it has reached the centre?

6. If BDA be a parabolic arc described by a comet round the Sun in the focus C, and P equal the periodic time of the Earth, her mean distance being 1; then the time of the comet's describing the arc $BDA = \frac{P}{12\pi}\{(a+b+c)^{\frac{3}{2}} - (a+b-c)^{\frac{3}{2}}\}$.

7. If two bodies moving, the one in a curve round, and the other in a straight line passing through the centre of force, have equal velocities at any the same distance from the centre, they will have equal velocities at all other equal distances.

8. If the force vary as $\frac{1}{(\text{Dist.})^2}$,

a = distance of projection from the centre,

P = perpendicular on this direction,

velocity of projection = velocity in a circle at that distance $\times\ q\sqrt{2}$,

x = radius vector,

θ = angle described.

Then prove that $d\theta = \frac{Pqdx}{x\sqrt{\{ax - (1-q^2).x^2 - p^2q^2\}}}$ and shew that this is an equation to a conic section.

9. In Prop. 45 it is said, that orbits have the same figure when the forces by which they are described are made proportional at equal distances: Prove this; and shew that no curve described by a force varying as any power of the distance, can have more than two different values of the apsidal distance.

10. If the force be constant, and the eccentricity of the orbit indefinitely great, prove that the angle between the apsides $= 90°$.

11. Prove that the time down any arc of a hypocycloid, beginning from the highest point, is proportional to the arc of the generating circle which is cut off by the string.

12. If a body, urged by a centripetal force, move on a curve surface whose axis passes through the centre of force, and if the path described by the body be projected on a plane perpendicular to the axis, shew that the projected area is proportional to the time.

13. If a body oscillating in a very small circular arc, have a very small motion communicated to it, when it is at its highest point, in a direction perpendicular to the plane of vibration, shew that it will describe a curve differing insensibly from an ellipse, and that its oscillations will be very nearly isochronous.

14. Let P and p denote the periodic times of a planet, and its satellite respectively s the sine of the angle under which, at the planet's mean distance from the Sun, the mean radius of the satellite's orbit is seen, then the quantity of matter in the planet is equal to $s^3 . \frac{P^2}{p^2} + \left(s^3 . \frac{P^2}{p^2}\right)^2$ nearly, the Sun's mass being 1.

15. Find the effect of the Moon in disturbing the motion of the Earth round the Sun, and shew that the force, urging the centre of gravity of the Earth and Moon towards the Sun, follows much more nearly the law of the inverse square of the distance than that which urges either of those bodies.

16. From the mean angular motion of the nodes of the Moon's orbit, deduce those of the satellites of Jupiter.

17. If a corpuscle be placed any where within a hollow cylinder, extended infinitely both ways in the direction of its axis, the attraction to each particle varying as $\frac{1}{(\text{Dist.})^2}$, prove that it will remain at rest.

18. If in an oblate spheroid of small eccentricity b be the polar radius, $b + c$ the equatoreal, and φ the angle which a semidiameter

makes with the axis; shew that the attraction on a point at the extremity of this semidiameter is equal to $\frac{4\pi b}{3}\left\{1+\frac{c}{5b}(4-\sin.^2\phi)\right\}$.

19. If a corpuscle of light, moving in a given direction, with a given velocity, be attracted towards a refracting medium terminated by a plane surface, by a force varying according to any power of the distance from it, find the equation to the curve which it describes, and shew that the sine of the angle of incidence is to the sine of the angle of refraction in a given ratio.

20. If the Earth were an homogeneous fluid mass, revolving round her axis, and gravity tended to the centre, and varied as the distance from it, prove on Newton's principle of all columns extending from the centre to the surface being in equilibrium, that her figure must be that of an oblate spheroid.

TRINITY COLLEGE, 1822.

1. Explain the principles of the methods of exhaustions, indivisibles, and limits. State accurately the meaning of the expression "ultimæ quantitates," as used by Newton; and show that the results he obtains by the system of limits are not approximately but strictly true.

2. Determine 1st., By the method of Exhaustions the value of the area of a circle. 2dly., By the method of Indivisibles the ratio between a sphere and its circumscribing cylinder. 3dly., By that of Limits the ratio between a cone and its circumscribing cylinder.

3. (1). Define accurately the circle of curvature.

(2). PT being the tangent to any curve PQ at P, and PV a straight line drawn, making any finite angle with PT, determine the curve FK whose intersection with PV in V, shall cut off PV a chord of the circle of curvature.

(3). Show that the direction with which F cuts or touches PV determines the degree of contact between PQ and its circle of curvature:—and that there may be an indefinite number of curves touching at P, to which the circle of which PV is a chord will be the circle of curvature.

(4). Prove that when PQ is a parabola, FK is a straight line.

4. "The areas round a centre of force are in the same plane, and proportional to the times." Prove the proposition. Explain the difference which in the change from polygonal to curvilinear motion takes place in the effect of the centripetal force;—and show that, notwithstanding this difference, BV, (the diagonal of the parallelogram AC,) may still be used as a proportionate measure of it.

5. At similar points in similar curves described round a centre of force similarly placed, $F \propto \frac{AS}{P^2}$, P being the periodic time.

6. When $F \propto \frac{1}{(\text{dist.})^2}$, the latera recta of the orbits described round it will $\propto \frac{(\text{areas d. t.})^2}{\text{abs. force}}$, and

$$\text{the velocities} \propto \frac{\sqrt{(\text{lat. rect.} \times \text{abs. force})}}{\perp^r \text{ on the tangent}}.$$

7. Investigate the equation $F = \frac{V^2}{m \, . \, PV}$, (where $m = 16\frac{1}{12}$ feet), and explain to what units F and V are referred in this and similar equations.

8. Prop. x. Cor. 1. "Si vis sit ut distantia movebitur corpus in ellipsi centrum habente in centro virium." Given the velocity and direction of a projectile attracted by such a centre of force, construct the curve it will describe, and show from the construction that whatever be the velocity, the curve will still be an ellipse.

9. Prop. xiii. Cor. 1. "Si corpus aliquod quâcunque cum velocitate exeat de loco P, et vi centripetâ simul agitetur quæ sit reciprocè ut quadratum distantiæ, movebitur hoc corpus in aliquâ Sectionum Conicarum umbilicum habente in centro virium." Construct the curve; and shew from the construction that according to the velocity of projection, the curve will be an ellipse, a parabola, or an hyperbola.

10. Determine the law of variation of the angular velocity in any curve; and compare the angular velocity at any point in the

ellipse, (force in the focus,) with the angular velocity in a circle at the same distance.

11. If an n^{th} part of the Earth were taken away, what change would be produced in the Moon's orbit? and in what ratio would her periodic time be increased? the orbit before the change being supposed circular. Exemplify in the cases where $n = 2$, or is greater or less than 2.

Conics.

1. If any two ordinates QQ', qq' terminated both ways by the curve of a parabola, intersect each other in M, and P, p be respectively their parameters, prove that $QM . MQ' : qM . Mq' :: P : p$.

2. In the ellipse, $CD^2 = SP \times HP$.

3. Rectangle under the abscissæ of any diameter to an ellipse : ord.^{2} :: $CP^2 : CD^2$.

4. Show that the hyperbola admits an asymptote:—and that, if any line Rr between the asymptotes cut the curve in R, r at a given angle, the rectangle $RP \times Pr$ will be invariable.

5. Any section of a paraboloid not perpendicular to the base is an ellipse.

6. Show that the shadow of the circular horizontal rim of a lamp traced out on a perpendicular wall is an hyperbola:—and the height of the flame above the rim, and its distance from the wall being given, construct the hyperbola, and determine its major and minor axes.

TRINITY COLLEGE, 1823.

1. If a right cone be cut by a plane of known inclination to the axis, find the equation to the section, and shew in what cases the section will be a circle, an ellipse, an hyperbola, or a parabola.

2. A parabola and an ellipse being traced upon a plane, find

   (1). The axis of the former.

   (2). The centre and axes of the latter.

3. Prove that the equation to an ellipse, when referred to its centre and axes, is

$$a^2 y^2 + b^2 x^2 = a^2 b^2.$$

And shew that when the co-ordinates are transferred to any system of conjugate diameters $2a'$ and $2b'$, the equation will become

$$a'^2 y^2 + b'^2 x^2 = a'^2 b'^2.$$

4. In the ellipse prove

(1). That $a^2 + b^2 = a'^2 + b'^2$.

(2). That tangents applied at the extremities of conjugate diameters will form, when produced to meet, a parallelogram.

(3). That the areas of all such parallelograms are equal.

5. Find the equation to the ellipse when referred to the focus and to polar co-ordinates; and thence deduce the polar equation to the parabola.

6. Given the equation to the hyperbola,

$$a^2 y^2 - b^2 x^2 = - a^2 b^2,$$

Shew by the transformation of co-ordinates that when the hyperbola is referred to its asymptotes, the equation becomes

$$xy = \frac{a^2 + b^2}{4}.$$

7. If two straight lines intersect within any conic section, prove that the rectangles contained by the corresponding segments will be to one another in a given ratio.

TRINITY COLLEGE, 1823.

1. STATE the principle on which is founded the doctrine of prime and ultimate ratios; and determine the ultimate ratio of an hyperboloid to its circumscribing cylinder.

2. The homologous sides of similar curvilinear figures are proportional, and the areas are in the duplicate ratio of the homologous sides.

3. Prove that the chord, the arc, and the tangent, in curves of continuous curvature, are ultimately equal.

4. Define force, absolute force, accelerating force, centripetal, and centrifugal force; periodic time. What is meant by the periodic time of a body moving in a spiral?

5. The velocities at any points in the orbit of a body, acted on by a centripetal force, being given, to find the centre of force.

6. (1). If a body describe any curve, and (p) be the perpendicular from the centre of force on the tangent at the distance (r), prove that the centripetal force $= h^2 \cdot \frac{dp}{p^3 dr}$: ($h$) being twice the area described in the unit of time; hence

(2). Find the value of the force in the ellipse and hyperbola about the focus.

7. The direction of the force by which a body describes a parabola being perpendicular to the axis, find the law of force.

8. A body being supposed to move in the logarithmic spiral,

(1). Find the space PL through which it must fall, by the action of the force at P continued uniform, to acquire the velocity at that point, and

(2). Shew that the locus of the point L is also a logarithmic spiral.

9. Compare the velocity of a body moving in any curve with its velocity in a circle at the same distance; and prove that at the point where these velocities are equal the angle contained between the tangent and radius vector is a minimum.

10. If a body describe an ellipse by a force tending to the centre,

(1). Find the law of force.

(2). The actual value of the periodic time;

(3). The point at which the centripetal and centrifugal forces are equal: and prove that the paracentric velocity at the same point is a maximum.

11. If a body describe an ellipse about the focus, and at any point be projected with its velocity at that point in the contrary direction to the force, determine how far the body will recede from the centre.

12. A body being projected from a given point with a given velocity in a given direction, and acted on by a given force varying as $\frac{1}{(\text{dist.})^2}$, find the elements of the curve which is described.

13. The force by which a body describes an ellipse being suddenly altered in any given ratio; required the alteration produced in the orbit.

14. (1). The path of a body moving in a plane, and acted on by forces in that plane, being referred to rectangular co-ordinates, shew that the equations of the body's motion are

$$X = \frac{d^2x}{dt^2}, \text{ and } Y = \frac{d^2y}{dt^2},$$

X and Y denoting the sum of the forces in the directions (x) and (y) respectively.

And thence prove

(2). That when the body is acted on by a central force, the sectorial areas described are proportional to the times.

TRINITY COLLEGE, 1824.

1. A TANGENT to a parabola at any point bisects the angle contained between the focal distance, and a perpendicular on the directrix.

2. If any two chords of a parabola intersect each other, either within or without the curve, the rectangles under their segments are to each other in the same ratio as the rectangles under the segments of any other chords which are parallel to the former.

3. If PG be a diameter in any ellipse; CP, CD semi-conjugates, and QV a semi-ordinate to PG,

$$PV . VG : QV^2 :: CP^2 : CD^2.$$

4. If PF be a perpendicular from P on CD;

$$PF . CD = AC . BC.$$

5. If n be a normal to any ellipse whose semi-axes are a, b; and λ the angle at which it cuts the major-axis,

$$n = \frac{b^2}{\{a^2 . \cos.^2 \lambda + b^2 . \sin.^2 \lambda\}^{\frac{1}{2}}}.$$

6. In any conic section, the radius of curvature is equal to the cube of the normal divided by the square of half the latus rectum.

TRINITY COLLEGE, 1824.

1. Describe the experiment upon the collision of bodies given in the introduction, which tends to establish the law according to which moving force communicates motion.

2. State the arguments which Newton employs to defend his method of prime and ultimate ratios, in the Scholium to the first Section.

3. When a body describes a curve by the action of a force proceeding from a fixed centre, the areas described by the line connecting the body with the centre, are in the same fixed plane, and are proportional to the times.

4. If v be the velocity of a body at any point acted upon by any number of forces F, F', &c. in the same plane, and c, c', &c. be the chords of the circle of curvature drawn through the centres, then will $v^2 = \frac{1}{2} Fc + \frac{1}{2} F'c' +$ &c.

5. To find the forces which must act upon a point, so that it may describe the arc of a parabola with a uniform motion.

6. Calculate the greatest possible height to which materials could be piled up above the surface of the Earth, in any given latitude.

7. Construct for the place of a comet in its parabolic orbit at any assigned distance of time from the epoch of its being in the perihelion.

8. Investigate an expression for the velocity of a body moving in an ellipse round the focus; and shew, from the nature of the result, that if another body begin to move freely towards the centre, from a distance equal to the axis major, its velocity will always be equal to that in the conic section at the same distance from the focus.

9. Having given the velocity, distance, and direction of projection, and supposing the force to remain the same at all distances from the centre, apply the principles of the general proposition in the eighth Section, to find the equation of the radius vector and perpendicular upon the tangent in the trajectory described.

10. Deduce the apsidal equation of the curve in the last question by making $ABFD - Z^2 = 0$: Prove that it has three real roots, and shew to what part of the trajectory the negative apsidal distance must belong.

11. Shew that a body may be made to move in an orbit revolving round a centre of force in the same manner as another body moves in a similar and equal orbit at rest.

12. Give all the steps of the method by which Newton proposes to trace the path of a body moving on a curve surface, round a centre of force situated in the axis.

13. If any number of bodies be acted upon only by their mutual attractions, their centre of gravity will either be at rest, or will move uniformly in a straight line; and the whole momentum of the system estimated in a given direction will always remain the same.

14. Suppose a central body to attract several others, and to be attracted by them, according to the law of gravity, but that the mutual attractions of the bodies are neglected. If also, the bodies be all equal, or exceedingly small, compared with the central mass, and are launched from one point with equal velocities, but in any different directions, prove

(1). that they will all describe the same kind of conic section;

(2). the sections will all have the same axis major;

(3). if the bodies do return, there will be a general collision of them all, at the end of the same given time.

15. In a given position of the system of the Sun, Earth, and Moon, investigate expressions for

(1). the whole force in the direction of the radius vector of the Moon's orbit round the Earth;

(2). the force perpendicular to the radius vector;

(3). the force which acts at right angles to the plane of the Moon's orbit.

16. If P and p be the periodic times of the Earth and Moon, and u be the Moon's velocity in quadratures; her velocity v, when at the angular distance θ from quadratures, may be found as a first approximation from this proportion, $v : u :: 3p^2 \sin.\theta^2 + 2P^2 : 2P^2$.

17. Prove that whenever the disturbing force tends to elevate the Moon from the plane of the ecliptic, the node advances, and in every other case retreats; but that the preponderating tendency of the node on the average of a whole revolution is always in favour of its retreat.

18. The precession of the equinoxes is explicable on the same principles as the motion of the Moon's nodes.

19. Supposing an error of 1′ in the Moon's distance from a star, as given by the tables, what would be the corresponding error in longitude at the equator? Enumerate the principal equations to the Moon's motion, which have been furnished by the theory of universal gravitation.

20. If equal centripetal forces tend to all the points of a spherical superficies, but decrease as the squares of the distances increase; prove that a particle situated within the superficies will remain unaffected by their attractions.

21. Calculate the deviation of the plumb-line from the vertical, when a particle at its extremity is attracted to a given adjacent spherical mass at the Earth's surface, the law of attraction being that of the inverse square of the distance.

22. If two similar mediums are separated from each other by a space terminated on each side by parallel planes, and a body in its transit through this space is attracted or impelled perpendicularly towards either medium, and is not agitated or hindered by any other force; and the attraction is every where the same at equal distances from either plane, taken towards the same side of the plane, prove that the velocity of the body before incidence is to its velocity after emergence, as the sine of emergence to the sine of incidence.

TRINITY COLLEGE, 1826.

1. DEFINE similar curves, and prove catenaries to be similar.

2. A body urged by a central force describes an arc of a curve: find the velocity of the point in which the radius vector intersects the chord of this arc; and determine at what point this velocity is a maximum and minimum.

3. The times of describing all parabolic arcs which are cut off from different parabolas having a common focus by chords passing through it, are to one another in the sesquiplicate ratio of those chords.

4. Find how far a body must fall externally to acquire the velocity in an ellipse, both when the centre of force is in the focus, and when it is in the centre of the ellipse.

5. If a circle be always described passing through the place of a comet in its parabolic orbit, and through the vertex and focus, prove that the centre of this circle will have an uniform motion along a line which bisects at right angles the perihelion distance. Determine also the velocity of this motion.

6. A body projected in a given direction, with a given velocity, and acted on by the interrupted impulsive action of a deflecting force, describes equal areas in equal times round a certain point: prove that the force tends to this point, and determine its measure, supposing the force to act not at intervals but continuously.

7. Prove Prop. 39, and its Corollaries.

8. If the force $\propto \frac{1}{\text{Dist.}^3}$, and the velocity of projection be greater than that in a circle, find within what limits the angles of projection must lie that the curves may continue of the same species.

9. The difference of the forces by which a body may be made to move in the quiescent and in the moveable orbit $\propto \frac{1}{\text{Dist.}^3}$ from the centre.

10. If the force $\propto \frac{bA^m \pm cA^n}{A^3}$, find the angle between the apsides in orbits nearly circular.

11. Find the time of a body oscillating in a hypocycloid; and hence deduce the time of oscillation in a common cycloid.

12. A sphere fixed to the extremity of a rod of inconsiderable thickness oscillates in a circle round its other extremity, the sphere being acted on by a force which $\propto \frac{1}{\text{Dist.}^2}$, and tends to a point within the circle; find the time of a small oscillation.

13. Find the deflection of the Moon in 1″ of time towards the Earth, taking into account the Sun's disturbing force, the quantity of matter in the Moon, and the centrifugal force arising from the rotation of the Earth round its axis.

14. Two spheres composed of concentric layers of different density will attract one another with moving forces, which vary as the product of their masses directly and as the squares of their distances inversely.

15. Shew that the mean value of the addititious force is nearly $\frac{1}{179}$ of the Moon's gravity.

16. Shew that the problem of the three bodies depends for its solution on the integration of the equation

$$\frac{d^2u}{d\theta^2} + u + \frac{T\frac{du}{d\theta} - Pu}{u^3\left(h^2 + 2\int\frac{T}{u^3}d\theta\right)} = 0;$$

and give the different steps of the solution and the mode of excluding all terms which are not periodical from the value of the radius vector.

17. Explain Clairaut's method for determining the progression of the lunar apogee, the cause of his first erroneous result, and the method of correcting it.

18. Shew that there are two forms of a fluid spheroid revolving round its axis which are those of equilibrium.

TRINITY COLLEGE, 1827.

1. STATE the nature of the angles of contact supposed by Newton in his reasonings of the 1st Section, and shew that between a common parabola and its tangent, an infinite series of parabolas of a higher order may be inserted; each being infinitely nearer to the tangent, towards the point of contact, than the one which precedes it.

2. The spaces which a body describes by the action of a finite force are, at the first instant of motion, in the duplicate ratio of the times. Prove this, and point out the case in which the proposition is true without the limitation.

3. Having given the force by which a body revolves in a given circle round a given point within it, determine the force by which a body may describe the same circle round another given point, in a period which bears to that of the first a given ratio.

4. Calculate from Newton expressions for the velocity at any point of the orbit, when a body revolves in an ellipse, hyperbola, or parabola.

5. If a body be projected in any direction with a given velocity, and acted on by a central force, which is as some function of the distance, its velocity at any distance is independent of the path described.

6. A body moves upon a plane curve; find the pressure upon it.

7. Find the least velocity which will cause a body to reach the Moon if projected directly towards it from the Earth, and acted on by lunar and terrestrial gravity.

8. Having given the radius of the Earth, considered spherical, the time of its rotation round its axis, and primitive gravity at its surface; find the sensible gravity at any point whose latitude is λ. Also the centrifugal force at the equator being now $\frac{1}{289}$th part of primitive gravity; find how much the rotation of the Earth must be increased in order that bodies in latitude 60° may cease to gravitate towards it.

9. Find the force by which a body may be made to describe a lemniscata, whose equation is $u^2 = a^2 . \cos.2\theta$, round a centre of force

in the nodus, and shew that the time of describing one of the ovals $= a^4 \sqrt{\frac{3}{\mu}}$, where $\mu =$ force at distance $= 1$.

10. A body is projected with a velocity equal to that from infinity, and acted on by a force which varies as $\frac{1}{D^n}$: determine the nature of the figure described, when n is greater, and also when less than 3.

11. Explain fully the method employed by Newton to determine the motion of the apsides in orbits which are very nearly circular. Prop. 45.

12. If the angle between the apsides $= 180° \sqrt{\left(\frac{b+c}{mb+nc}\right)}$; prove the force to vary as $\frac{bA^m + cA^n}{A^3}$.

13. A body acted on by gravity moves on the surface of a sphere in a path which is nearly a circle. Find the horizontal angle between the apsides.

14. A body is attracted towards a centre of force whose intensity varies as the distance. Find all the tautochronous lines.

15. Two bodies M and m, consisting of equal particles of matter, attracting each other with forces which are inversely as the squares of the distance, move round each other, affected only by their mutual gravitation:

(1). Explain the circumstances of their motion.

(2). Shew that the time of m's revolution $= \frac{2\pi a^{\frac{3}{2}}}{\sqrt{(M+m)}}$, where $a =$ the mean distance of m's orbit.

16. Shew the effects produced by the disturbing forces on the excentricity of P's orbit, when the apsides are in quadratures, and when they are in syzygies. Prop. 66, Cor. 9.

17. If the force on a body describing an ellipse round the focus be increased or diminished by a small quantity; find the alteration produced in the elements of the orbit, and in the periodic time.

18. If $F \propto \frac{1}{D^2}$, and a particle of matter be placed without a spherical superficies, it will be attracted towards the centre with a force which is proportional to the inverse square of the distance from it. Prop. 90.

19. The semi-axes of an oblate spheroid of small excentricity are b and $b + c$, and the angle between the major axis, and a line drawn from the centre of the spheroid to a point on its surface $= \phi$. Shew that the attraction of the spheroid on a particle placed in that point $= \frac{4\pi b}{3}\left(1 + \frac{c}{b} \cdot \frac{4 - \sin.^2\phi}{5}\right)$.

20. Find the deviation of the plumb-line from the vertical, when a particle suspended at its extremity is attracted towards a given homogeneous sphere; the length of the pendulum being inconsiderable compared with the distance, which is given, between the attracting sphere and the point of suspension.

Conic Sections.

1. A CONE is cut by a plane. Determine the cases in which the section will be a parabola, an ellipse, and a hyperbola respectively.

2. To find the focus of a given parabola.

3. The perpendiculars from the foci upon any tangent to an ellipse intersect the tangent in the circumference of a circle whose diameter is the axis major. Prove this; state the corresponding proposition with respect to the parabola, and shew that it may be deduced from it.

4. In the ellipse the rectangle by the normal PG and the perpendicular PGF upon the conjugate diameter is equal to the square of half the minor axis.

5. If any number of tangents be placed round an ellipse so as to form a polygon, the continued products of the alternate segments of the sides, made by the points of contact, are equal.

6. If tangents be drawn at the vertices of four conjugate hyperbolas, the diagonals of the rectangle so formed are asymptotes to the four curves.

TRINITY COLLEGE, MAY 1831.

1. THE straight line, which bisects the angle contained by the radius vector SP, and the perpendicular from P on the directrix, is a tangent to the parabola.

2. In a parabola, the rectangle contained by the parameter to any diameter and the abscissa, is equal to the square of the ordinate.

3. If the normal to any point P of a parabola be produced to meet a perpendicular to the axis at the extremity of the subtangent in the point O, then will PO be equal to the radius of curvature at P.

4. In an ellipse the rectangle contained by the abscissas of the axis major is to the square of the ordinate, as the square of the axis major to the square of the axis minor.

5. In the ellipse $SP \times HP = CD^2$.

6. In the ellipse, if there be drawn two straight lines KL and MN, intersecting one another at right angles in the point O, and parallel to the axis major, and axis minor respectively; then will

$$KO^2 + LO^2 + MO^2 + NO^2 = 2AC^2 + 2BC^2.$$

7. In the hyperbola, the semi-axis major CA is a mean proportional between the abscissa CM, and the distance CT.

8. If a straight line touching an hyperbola in a point P, be produced to meet the asymptotes in K and L, the triangle CKL is a constant magnitude.

9. The sections of two similar and concentric and similarly situated spheroids, made by the same plane, are similar and concentric ellipses, similarly situated.

10. Prove that the ultimate ratio of the arc, chord, and tangent is the ratio of equality. Is, or is not this already assumed as true for similar figures?

11. A body projected from a given point, in a given direction, and with a given velocity, will, if referred to another given point, describe the same area in the same time, whether it be, or be not acted on, by a force tending to that point.

12. Prove the following expressions for the force acting on a body moving in a curve:

(1). $f = \frac{2QR}{T^2}$

(2). $f = \frac{2h^2 . QR}{SP^2 . QT^2}$, where $h = 2$ area in 1″

(3). $f = \frac{2h^2}{SY^2 \times PV}$

(4). $f = \frac{2v^2}{PV}$

13. Apply the second of the above expressions when a body revolves in an ellipse both round the centre, and round the focus.

14. If the force $\propto$ distance, a body will describe an ellipse, or a circle whose centre coincides with the centre of force. Cor. 1. Prop. x.

15. Find an expression for the angular velocity in any curve; and explain the nature and origin of what is called centrifugal force: deduce also this expression for it's value, viz. $\frac{h^2}{SP^3}$

16. The velocity in any conic section, the force being in the focus, is $\frac{\sqrt{(\frac{1}{2} lm)}}{SY}$, where l = latus rectum.

17. Give Newton's construction for determining the conic section that will be described, when the force $= \frac{m}{\text{dist.}^2}$, and a body is projected from a given point, in a given direction and with a given velocity. Prop. XVII.

18. Investigate the equation $\frac{d^2u}{d\theta^2} + u - \frac{P}{h^2u^2} = 0$, and apply it to determine the law of force in a circle, the centre of force being in the circumference.

TRINITY COLLEGE, May 1831.

1. How far must a body fall externally to acquire the velocity in the three conic sections, the force being in the focus?

2. Prove that the time of describing the angle ASP in an ellipse is equal to

$$\sqrt{\left(\frac{2(1+\beta)}{m}\right)} D^{\frac{3}{2}}\left\{\left(t+\frac{t^3}{3}\right)-2\beta\left(\frac{t^3}{3}+\frac{t^5}{5}\right)+\frac{2.3}{1.2}\beta^2\left(\frac{t^5}{5}+\frac{t^7}{7}\right)-\&c.\right\}$$

where m = force at distance 1, $D = AS$, $\beta = \frac{1-e}{1+e}$, and $t = \tan.\frac{ASP}{2}$.

3. If the force $\propto \frac{1}{\text{dist.}^3}$, and the velocity of projection be greater than that from infinity, give Newton's construction for the curve.

4. If the force $\propto \frac{1}{\text{dist.}^n}$, and the velocity of projection be that from infinity, find the equation to the curve described.

5. Explain fully Newton's method of finding the angle between the apsides in orbits nearly circular: and apply it to the case where the force $= \frac{bA^m \pm cA^n}{A^3}$. What is the reason of the difference of the results according as T is, or is not taken equal to 1?

6. If the force $\propto$ dist.^{n}, and a body be projected from an apsidal distance R, with a velocity such, that its square $= (1-e) \times$ velocity^2 in a circle, the equation to the curve described will be

$$\theta = \frac{1}{\sqrt{(n+3)}} \text{arc}\left(\text{vers.} = \frac{n+3}{Re}x\right)$$

where $x = R - r$, and x and e are very small quantities of the same order; and the squares and products of them are neglected.

7. To find the time in which a body oscillates in a given hypocycloid.

8. If a body be projected from a given point, with a given velocity, and be acted on by forces X, Y, Z in the direction of three rectangular co-ordinates; then will the velocity at any other point

be the same, whatever be the path described, provided

$$Xdx + Ydy + Zdz$$

be an exact differential of a function of three variables.

Shew that this will be the case when the forces acting on the body tend to fixed centres, and vary as any functions of the distances.

9. If a body revolve in an elliptic groove acted on by forces tending to the foci, and to the centre, the two former varying inversely as the square of, and the third directly as the distance: find the pressure at any point. Prove also that if the square of the velocity of projection be equal to the sum of the squares of the velocities which the body ought to have to revolve freely round the three centres of force taken separately, it will revolve round them freely when taken conjointly.

10. If a body be projected, at a small angle of elevation, in a medium where the resistance varies as (velocity)^2, and gravity be constant, find the equation to the curve described.

11. Find the law of resistance and density, that a body acted on by a force tending to a fixed centre, and varying according to any power of the distance, may describe a given curve. Apply the result to the ellipse, the centre of force being in the focus.

12. Let there be two concentric spheres, the particles of which attract with forces varying as any power of the distance; then will the attraction of the inner sphere on any point in the surface of the outer : attraction of the outer sphere on any point in the surface of the inner :: surface of the inner sphere : surface of the outer.

Also the actions of the entire spheres on the surfaces of one another will be equal.

13. Find the attraction of the matter contained between the surface of the Earth and its inscribed sphere on points in the polar and equatorial axis: the Earth being supposed to be an homogeneous oblate spheroid of small eccentricity.

14. If q denote the ratio of the centrifugal force at the equator to the force of gravity, prove that whatever be the law of the Earth's density, its ellipticity must lie between the limits $\frac{q}{2}$ and $\frac{5q}{4}$. Shew also that Newton was mistaken in supposing that a greater density

towards the centre would be accompanied with a greater degree of oblateness.

15. If the attraction to every particle of a sphere $\propto \frac{1}{\text{dist.}^4}$, then will the attraction of a point which is at a great distance from it in comparison with the radius of the sphere, also $\propto \frac{1}{\text{dist.}^4 \text{ from the centre}}$ very nearly.

16. Investigate expressions for the disturbing forces of the Sun on the Moon, their orbits being supposed to be in the same plane.

17. Let $n = \frac{\text{sidereal month}}{\text{sidereal year}}$,

$\theta =$ longitude of the Moon,

$1 =$ force of gravity on the Moon,

$1 =$ mean radius of the lunar orbit,

$N =$ longitude of the ascending node,

$\theta_1 =$ horary motion of the Moon,

then will the horary motion of the ascending node

$$= -3n^2\theta \,.\, \sin.(\theta - N)\sin.(n\theta - N)\cos.(1-n)\theta\,;$$

and the mean motion of the node will

$$= -\frac{3n^2}{4}\theta\left\{1 - \frac{3n}{2(4+3n)}\right\}$$

18. Prove that the following property belongs exclusively to the law of nature, where gravitation $\propto \frac{\text{mass}}{\text{dist.}^2}$; viz.

That if the magnitudes of all the bodies in the universe, their mutual distances and velocities were increased or diminished in the same proportion, they would still describe curves similar to those which they describe now, and the appearances would be in every respect exactly the same.

TRINITY COLLEGE, June 1832.

1. If P be a point in a parabola, and QQ' be drawn parallel to the tangent at P, and PV parallel to the axis, QQ' is bisected in V.

2. A parabola being traced upon a plane, find its axis.

3. If a right cone be cut by a plane through both slant sides, the section is an ellipse.

4. If PG be a diameter in any ellipse, CP, CD semi-conjugates, QV a semi-ordinate to PG, and PF a perpendicular upon CD; prove that $PV \times VG : QV^2 :: CP^2 : CD^2$;

also, that $PF \times CD = AC \times BC$.

5. If tangents be drawn at the vertices of four conjugate hyperbolas, the diagonals of the rectangle thus formed are asymptotes to the curves.

6. If PQ be an arc of any conic section, QR parallel and QT perpendicular to the radius-vector, the limit of $\frac{QT^2}{QR}$ = the latus rectum.

7. Prove *strictly* Newton's fourth Lemma, establishing the previous principles.

8. Define *finite curvature;* and show that in such curves, the subtense is ultimately as the square of the conterminous arc.

9. If a body, moving in a curve, describes round a fixed point areas proportional to the times, it is acted upon by a force tending to that point; but if the areas *increase*, the direction of the force is turned to the side towards which the body is moving.

10. If F and f be the forces, D and d the central distances at similar points of similar curves, P and p the times of describing similar portions round centres of force similarly situated, then

$$F : f :: \frac{V^2}{D} : \frac{v^2}{d} :: \frac{D}{P^2} : \frac{d}{p^2}$$

Hence, if $P^m \propto V^n$, find the variation of the force.

11. A curve being given, which is described by the action of a central force, the actual value of the force may be found by determining the ultimate value of the quantity $\frac{8A^2 . QR}{SP^2 . QT^2}$, A being the area described in a unit of time.

12. Apply this,

(1). To the case of a body moving in a cycloid, force acting in lines parallel to the axis.

(2). To the case of a body revolving in the equiangular spiral.

13. A body revolves in an ellipse; it is required to find the law of force tending to the focus.

14. A given force being inversely as the square of the distance from the centre, and the velocity and direction of the motion at a given point being known; to determine the curve described.

Find the change in the orbit, when the force acting on the body at any point is altered.

TRINITY COLLEGE, June 1832.

1. If a body revolve about another body which is moving in any manner whatever, and if the first body describe about the second areas proportional to the times, the first body is acted on by a centripetal force tending to the second body, and also by the whole of the force by which the second body is acted on.

2. State Kepler's three laws. By what means did he arrive at that of the equable description of areas?

3. How did Newton infer that the Moon was retained in her orbit by the force of gravity? Perform the calculations, which shew it, approximately.

4. A body describes a parabola whose equation is $y^2 = 4mx$, the velocity in the direction of the ordinates being constant; determine the velocity at any point, and the force.

5. Compare the velocity which a body would acquire in falling from the Moon to the Earth with that which it would acquire in falling from infinity to the same point, the distance of the Moon from the Earth's centre being 60 radii of the Earth, and the force varying inversely as the square of the distance from that centre. Find also the time of falling to the Earth.

6. Find the time of describing a given true anomaly (1) in a parabolic orbit, (2) in a very eccentric ellipse.

7. When a body describes a curve by the action of a central force, the polar equation to the orbit is $\frac{d^2u}{d\theta^2} + u - \frac{P}{h^2u^2} = 0$. Also the velocity at any point is independent of the nature of the path

described. The same is true of constrained motion on a curved surface.

8. While a body is revolving uniformly in a circle, acted on by a force varying as the distance, the force is suddenly doubled by the accession of a new force varying as $\frac{1}{(\text{dist.})^3}$; determine the equation to the curve described, and the time of describing a given angle.

9. Apply the equations of motion to determine the circumstances under which a body acted on by given forces, will uniformly describe a circle on a surface of revolution.

10. What does Newton prove in the two first Propositions of the ninth Section? Explain fully his mode of determining the motion of the apsides in orbits nearly circular, and apply it to the case where the force $\propto \frac{1}{a^2 + (\text{dist.})^2}$.

11. The orbit which a planet appears to describe round the Sun, or the Sun about a planet, is an ellipse. Shew this, and determine the period.

12. If the motion of a body P revolving about another T, be disturbed by the action of a very distant body S in the plane of P's orbit, the disturbing forces on P in directions ST, and perpendicular to ST, are nearly as $2PK$, and PL, where PK and PL are perpendiculars from P on the lines of quadrature and syzygy.

13. Shew also that the tangential disturbing force is nearly $k \sin.2\theta$, where θ is the angular distance of P from quadratures; and if k be so small that its square may be neglected, and p be the period of P in its circular orbit round T, and $PT = r$, the time of describing θ will $= \frac{p}{2\pi}\left(\theta + \frac{kp^2}{16\pi^2 . r} \sin.2\theta\right)$.

14. If S be not in the plane of P's orbit, explain generally how a motion of the nodes, and a variation of the inclination of P's orbit round T will ensue. How far do these effects go to prove the law of universal gravity?

15. Determine the attraction of a spherical shell of indefinitely small thickness on a particle either within or without it, attraction $\propto \frac{1}{(\text{dist.})^2}$.

16. Find also the attraction of a spherical sector on a particle placed at the vertex; and if the quantity of matter be given, shew that the attraction is the greatest when the angle of the sector is $= 2 \cos.^{-1} \frac{1}{5}$.

17. A body acted on by gravity is projected in a medium of which the resistance varies as the velocity; compare the velocity of the body at the expiration of any time with that which it would have had *in vacuo*.

18. If the Earth's surface were covered with water, why would the figure assumed by the waters be nearly that of a prolate spheroid?

19. Eliminate t and ρ from the equations

$$(1).\quad \frac{d}{dt}\left(\rho^2.\frac{d\theta}{dt}\right) = T\rho$$

$$(2).\quad \frac{d}{dt}\left\{\left(\frac{d\rho}{d\theta}\right)^2 + \rho^2\left(\frac{d\theta}{dt}\right)^2\right\} = -2P\frac{d\rho}{dt} + 2T\rho\frac{d\theta}{dt}.$$

$$(3).\quad u = \frac{1}{\rho}$$

What is the nature of the method pursued to integrate approximately the resulting equation between u and θ?

St. JOHN'S COLLEGE, 1814.

1. Define prime and ultimate ratios, and from your definition find the value of $\frac{X}{Y}$, X and Y being any functions of x, which vanish when $x = a$, and apply your result to find $\int \frac{x^{\cdot}}{x}$ considered as the limit of $\int x^n x^{\cdot}$ when $n = -1$.

2. Square the common parabola by Newton's fourth Lemma.

3. *Newton*, Lemma 11. The subtense of the angle of contact is ultimately in the duplicate ratio of the subtense of the conterminous arc.

4. If two or more centres of force be situated in one right line, the area described by the projection of any body round the projection

of the centres of force, on a plane perpendicular to this line, is proportional to the time.

5. If there be any system of bodies, acted on only by their mutual attractions and repulsions, and those of an immoveable centre of force S; the sum of the projections of the areas described round S on any plane, multiplied by the respective masses of the describing bodies, is proportional to the time.

6. Shew that the force $\propto \frac{p^{\cdot}}{p^3 x^{\cdot}}$, and if $F = c.\frac{p^{\cdot}}{p^3 x^{\cdot}}$, find c.

7. Define similar curves, as referred to two rectangular co-ordinates x and y. Shew that in the equation $(1 - e^2)(a^2 - x^2) = y^2$ the variation of a gives all the similar curves which this equation can represent. In general shew, that if the equation of a curve be reducible to the form

$$0 = \text{any function of } \left(\frac{x}{a}\right) \text{ and } \left(\frac{y}{a}\right),$$

the same holds good. From hence prove that Catenaries are similar. Define similar curves of double curvature.

8. Find, as Newton has done, the law of force to the centre of an ellipse.

9. Compare the centripetal and centrifugal forces, in general, and in the Lemniscate.

10. A body P, suspended by a perfectly elastic thread CP, oscillates freely, describing a certain curve ABD. Find the velocity at any point P, and shew that the momentary acceleration in the description of areas round C, is proportional to the ordinate PN.

11. The parameter of a conic section described by a force in the focus $\propto \frac{(\text{area dat. tem.})^2}{\text{absol. force.}}$

12. The force to a centre S being $= \frac{f}{x^2}$, x being the distance, a body is projected from a distance $SV = D$, at an angle θ, and with a velocity $= n \times$ velocity in a circle. Find the actual values of the following quantities, (1), the semiparameter; (2), the excentricity; (3), the semitransverse axis of the conic section described. Shew that the axis is independent on the direction, and the eccentricity on the distance of projection.

13. In an ellipse whose excentricity is e, shew that the relation between the angle $ASP = \theta$, and the time t of describing it is expressed by an equation $nt = \varpi - e. \sin. \varpi$, ϖ being dependent on θ by the equation

$$\tan. \frac{\varpi}{2} = \sqrt{\left(\frac{1-e}{1+e}\right)} \cdot \tan. \frac{\theta}{2}$$

14. *Newton*, Prop. 35. The area DES described by the revolving radius SD is equal to the area which a body would uniformly describe in the same time in a circle with a radius $= \frac{1}{2}$ the parameter of DES, round S. Give a proof in the case when DES is a parabola.

15. Force $\propto \frac{1}{x^n}$ and velocity = that from infinity. Find the actual equation between the distance $CP = x$ and the angle $VCP = \theta$, and shew that the curve VPK has always the following property

$$\left(\frac{n-3}{2}\right) \text{ times } \angle VCP = \angle PCQ.$$

Find also what curves are described when $n = 4$, and when $n = 7$, V being an apse.

16. In the 9th section, the revolving orbit is one of Cotes's spirals. Shew that the absolute orbit is a spiral of the same species.

17. Explain Newton's meaning, Prop. 61. "Motus eorum perinde se habebunt ac si non traherent se mutuo sed utrumque a corpore tertio in communi centro gravitatis constituto, viribus iisdem traherentur." Can one and the same body placed in C fulfil the condition here implied with respect to *both* the bodies S, P? Find C.

18. Required the whole effect, and also the mean effect, of the Sun, to diminish the Lunar gravity, and shew that if P, p, be the periods of the Earth and Moon, $f =$ the Earth's attraction, and r the radius vector of the Moon, the addititious force will be nearly represented by the formula

$$\left\{\left(\frac{p}{P}\right)^2 - \frac{1}{2}\left(\frac{p}{P}\right)^4\right\} \cdot fr$$

the Moon's mean distance being taken for unity.

19. The Moon's force to raise the tides is proportional to the cube of her parallax.

20. Investigate the figure of equilibrium of a revolving fluid mass; attraction varying as the distance of any two particles from each other, directly.

St. JOHN'S COLLEGE, Dec. 1815.

1. By what observations, and by what reasoning from them, do we conclude that the planets are retained in their orbits by a force which is inversely as the square of their distance from the Sun? and what reasons have we for supposing that $\frac{1}{(\text{dist.})^2}$ continues to represent nearly (or accurately) the variation of the force, up to a comparatively small distance from the surface of bodies?

2. Prove rigorously, that the ultimate ratio of vanishing quantities is the ratio of their fluxions; and find the ultimate ratio of the segment of a sphere to its inscribed cone. Prove also, that the same segment is to its least circumscribed cone (ultimately) as 8 : 9. What is the ultimate ratio of $x^x + \cos. \pi x : x - 1$, when x approaches to 1 as its limit?

3. Find the law of force tending in parallel lines, by which a body may be made to describe an hyperbola whose axis is parallel to the direction of the force.

4. Find, as Newton has done, the law of force by which a body may describe a given orbit; then apply your result to find the orbit when the force is given; and, as an example, determine the orbit when the force $\propto \frac{1}{(\text{dist}.)^4}$, and the velocity at any point is equal to that which would be acquired by falling down an infinite distance.

5. Suppose A to attract B with a force always double of that with which B attracts A; the bodies being originally at rest, shew that their centre of gravity will begin to move, and find its position, and that of A and B after any assigned time; the law of either force being the direct ratio of the distance between the bodies.

6. Two equal bodies are projected at the same instant from the equator and the pole, in the plane of the same meridian, and at an angle of 45° to the horizon. With what velocity must each be pro-

jected, so that after meeting the other in its course, it shall be reflected back to the point from which it set out, elasticity being perfect?

7. Define similar curves, and prove that all curves represented by an equation of the form $y^3 + y^2 x = a^3$ are similar. Prove by prime and ultimate ratios that similar solids are in the triplicate ratio of their homologous sides.

8. The centripetal force varying as $bA^{m-3} + cA^{n-3} + eA^{p-3}$, determine the angle between the apsides; and give the reasons for each step of the process.

9. A body sets out from one extremity of the chord of a circular arc, and, in the course of its revolution arriving at the other, is reflected directly into the centre. Compare the whole time of its motion with the time of a complete revolution.

10. In the last problem, find what arc of the circle the chord must subtend, so that after reflection the body shall describe an ellipse of a given excentricity; and prove that whatever arc it subtends, the period in the ellipse described will be the same.

11. A body (considered as a mere material point) is acted on by forces, whose quantities, and directions, and the laws of whose variations are given. How would you proceed to determine the curve it will freely describe in consequence of their action? Apply your reasoning to the case of a single force tending to a fixed centre.

12. Draw a diameter of a given ellipse in such a manner, that a planet revolving about its focus may describe the two segments of its orbit in times which are to each other in the proportion of $n : 1$; and shew that unless this be a less ratio than that of the sum of the circumference of a circle, and twice its diameter to their difference, no such line can be drawn.

13. Investigate an expression for that part of the ablatitious force which diminishes the Moon's gravity; the lunar orbit being supposed coincident with the ecliptic. How will this expression be affected by taking into consideration the inclination (supposed very small)?

14. In the Scholium to the first Section, Newton speaks of a series of angles of contact, each of which is infinitely less than the

foregoing. Explain his meaning. Find the latus rectum of a parabola whose axis is parallel to the abscissa of a given curve, and which shall touch the curve so closely in a given point, that no parabola (similarly situated) can pass between it and the curve, at that point.

15. Two bodies which revolve round their common centre of gravity, describe about it, and about each other, areas proportional to the times.

16. In an elliptic orbit, if u represent the eccentric anomaly, and the fraction $\frac{SC}{AC + BC}$ be represented by c, the excess of the true anomaly above the eccentric, will equal the double of the following series;

$$\frac{c}{1} \cdot \sin. u + \frac{c^2}{2} \cdot \sin.2u + \frac{c^3}{3} \cdot \sin.3u + \text{\&c. ad inf.}$$

St. JOHN'S COLLEGE, 1816.

1. Explain the connexion between the methods of prime and ultimate ratios, and of fluxions. What is the ultimate ratio of an hyperboloid to its circumscribing cylinder?

2. What is meant by continuous curvature, and how may we recognise the points where the curvature ceases to be so? What is signified by "a discontinuous curve?" What is Newton's meaning in the expression "circulum concentricé secat?"

3. Shew that $L = \frac{QT^2}{QR}$ in the hyperbola.

4. In an ellipse whose excentricity is small, the difference between two successive radii of curvature is ultimately as the square of the eccentricity.

5. Given the periodic times and mean distances of the primary planets and the mass of any one of them: compare the masses of the rest with that of the Sun.

6. In circles, when the period $\propto$ the radius ($P \propto R$) the centripetal force $\propto \frac{1}{R}$. If $P \propto R^2 + R + 1$, what is the law of force?

7. A body revolving in an ellipse about its centre, meets a perfectly elastic plane which produced would pass through the centre: what orbit will it describe after reflection?

8. (Newton, Prop. 60.) Compare the axis of the ellipse described by S and P round each other with that of the ellipse which P would describe round S fixed in the same period.

9. Find, from the necessary data, the absolute time (in seconds) in which a body will describe a given area of any curve, and apply your reasoning to the logarithmic spiral.

10. Find the value of Q in the 41st. proposition.

11. If one half of the Moon were *suddenly* deprived of gravity, what change would take place in her orbit, supposed at first a circle? If *gradually*, how would you proceed, were it required to estimate the effect?

12. Force $\propto$ Dist. Prove that the time of falling through any space : time of falling through the same space with the force at the greatest distance continued uniform :: arc : chord, of a circle whose radius is the greatest distance and versed sine the space fallen through.

13. The force $\propto \frac{1}{(\text{Dist.})^3}$, and the velocity at a given distance CP is greater than that in a circle. Take the angles CPA, CPB whose sine : rad. :: velocity in a circle : velocity of projection; then if a body be projected from P within the vertical angle BPa, or APb, Cotes's 3d spiral will be described; but if within either of the angles BPA, aPb, the 5th.

14. The velocity acquired in describing any curvilinear space is equal to the velocity acquired by falling freely in a right line to the same distance from the centre. Does any thing in Newton's proof of this proposition appear deficient, or superfluous, when applied to motion confined to a particular curve?

15. Can you prove the foregoing proposition (Newton, Prop. 40.) without, in any part of your argument, assuming the principle of the resolution of forces?

16. The force is inversely as the $(\text{Dist.})^2$. Give the full method of finding the elements of the orbit, having given the velocity, distance, and direction of projection.

17. Prove that $F \propto \frac{dp}{p^3 dx}$ and in the equation $F = c\ \frac{dp}{p^3 dx}$ find the value of c.

18. To what cases does Newton's proof of the formula $F \times PV, \propto V^2$ apply (Cor. 4. Prop. 6.)? Prove that, m being $16\frac{1}{12}$ feet, V^2 is equal to $m \cdot F \cdot PV$.

19. In the 9th Section, Newton's method gives very exactly one half of the true motion of the lunar apsides. Explain at some length why it is incapable of giving the whole, and give a brief outline of the train of argument used in that section.

20. In the 11th Section, shew that the force perpendicular to P's orbit $\propto \sin. A \cdot \sin. I \cdot \sin. Q$, where $A =$ the angular distance between the lines of nodes and syzygies, $I =$ the inclination, and $Q =$ the Moon's distance from quadrature, reduced to the plane of the ecliptic.

St. JOHN'S COLLEGE, June 1820.

1. Explain what is meant by continued finite curvature. Shew that if QP be any arc of a curve, and QR a subtense perpendicular to the tangent, limit $\frac{QP^2}{QR}$ = diameter of curvature at the point P; and apply this expression for finding the diameter of curvature at the vertex of a cycloid.

2. Let AB be any arc of a curve of finite curvature, AK, BK normals at A and B meeting in K, and BG perpendicular to the chord AB meeting AK in G; prove that in the limit $AK : AG :: 1 : 2$.

3. Investigate the relation between the centripetal and centrifugal forces at any point in any orbit: the equation to the curve in which they are equal; and the law of the force by which it will be described.

4. If the force $\propto \frac{1}{D^2}$ and a body descend in a straight line; find the velocity and time corresponding to any given space by Newton, Prop. 39. Cor. 2, 3.

5. If a body be projected in any direction from a given point above a given plane, and be acted upon by a force perpendicular to the plane, and varying inversely as the n^{th} power of the distance from it; find the equation to the trajectory, and shew for what values of n, that equation will be expressed in finite terms.

6. A body begins to fall from A to a centre of force S varying inversely as the cube of the distance; find the nature of the curve AP, when the time down AN is equal to the time of describing NP with the velocity acquired at N: AN being the abscissa.

7. Find generally the equation to the orbit in fixed space in Sect. 9. and from that equation, shew that the difference of force in the fixed and moveable orbit varies as $\frac{1}{D^3}$.

8. If the force vary as A^n, shew that the angle between the apsides in orbits nearly circular $= \frac{\pi}{\sqrt{(n+3)}}$ nearly, and when $n = 1$ explain the reason why we obtain an accurate result.

9. Find the nature of the curve which by its rotation round its axis will generate a surface, in which the times of revolution in circles parallel to the horizon shall be equal at all altitudes.

10. If a string will just bear (p) pounds; through what angle must it be made to oscillate with a weight (q) less than (p) at its extremity so that it may all but break?

11. Investigate an expression for the tangential force in P's orbit supposed circular, and find the velocity generated by it from quadrature to syzygy.

12. Find those positions of the apse of P's orbit where the eccentricity is a maximum and minimum, and explain fully Newton's reasoning in Cor. 9. Pr. 66.

St. JOHN'S COLLEGE, June 4, 1821.

1. If a line be drawn parallel to the base of a cycloid, find the limiting ratio of the segment of a cycloid to the corresponding segment of the generating circle.

2. Explain what is meant by angular velocity, and shew that in different ellipses of the same eccentricity round the same centre of force, $\left(F \propto \frac{1}{D^2},\right)$ the angular velocity at points which are at the same angular distance from the axis major $\propto \frac{1}{(\text{dist.})^{\frac{3}{2}}}$.

3. Determine the law of force acting in parallel lines perpendicular to the axis by which a body may be made to describe a parabola.

4. If a body projected obliquely be attracted by any number of bodies at rest ($F \propto D$), shew that it will describe an ellipse round their common centre of gravity.

5. *ABD*, *abd*, are two similar and concentric ellipses in the same plane, which revolves uniformly about their common centre *C*. Shew that if grooves *CaA*, *CbB*, be drawn from the centre, and bodies be placed at *a* and *b*, they will, by the motion of the plane, arrive at *A* and *B* in the same time.

6. Elasticity : perfect elasticity :: m : 1. A body *A* revolving in an ellipse $\left(F \propto \frac{1}{D^2}\right)$ strikes *B* at rest; find how the absolute force must be altered that *B* may describe the same orbit *A* was describing.

7. A stone suspended by a string which can support five times its weight, begins to describe a circle whose centre is *C* in a vertical plane. *AB* is a quadrant of this circle, *A* being the highest point. Find at what point *D* of *CB* an obstacle must be opposed, that the string may all but break when the body has acquired its greatest velocity.

8. *AP* is a parabola, *A* the vertex, *S* the focus, $SP = \frac{\text{lat. rect.}}{2}$. A perfectly elastic body descending from infinity toward S $\left(F \propto \frac{1}{D^2}\right)$ is reflected by a plane touching the parabola in *P*. Compare the time of its reaching *SA* produced with the time of a body's describing *PA* in the given curve.

9. In Prop. 66. shew that the effect is the same whether the sum of the disturbing forces be referred to P revolving round T at rest, or P and T revolve about their common centre of gravity acted upon by their respective disturbing forces.

10. If two bodies S and P attracting each other be projected in opposite and parallel directions, with velocities which are inversely as the quantities of matter, they will describe similar orbits round the common centre of gravity. Would this be the case if they were projected with any velocities?

11. The attractions of particles similarly situated with respect to similar solids are proportional to any homologous lines in those solids; the law of attraction being the inverse square of the distance.

St. JOHN'S COLLEGE, May 31, 1822.

1. Define similar figures when referred to an axis, and prove from the definition, that if ABC be a curve, AC the axis, and in every chord AD or AD produced, Ad be taken to AD in a given ratio, the locus of d will be a curve similar to ABC.

2. A body is moving in a curve in a direction making an angle of 30^0 with the distance. Supposing an impulse communicated to it, so as to make it move in a direction perpendicular to the distance with double its former velocity, compare the areas described in equal times round the centre of force.

3. Prove the ninth lemma.

4. A body revolves in a circle, the force tending to a point which is not the centre of the circle. Find the distance at which the angular velocity = the mean angular velocity.

5. If AB, ab be finite similar arcs of similar curves, S and s the centres of force similarly situated in them, P and p the times of describing AB, ab respectively; prove that the force at A : force at a :: $\frac{AS}{P^2} : \frac{as}{p^2}$; and point out in the demonstration the parts which depend on the suppositions respecting AB, ab, and the positions of S and s.

6. If $F \propto D$ and ellipses be described on the same axis major; prove that the times of moving from the vertex to a line drawn from any point in the axis perpendicular to the axis, will be equal.

7. A body descends from A towards S the centre of force $\left(F \propto \frac{1}{\text{dist.}^2}\right)$. ADS is a semi-circle, whose diameter is AS, and BD, CE are perpendicular to AS. Draw the straight line SFD, then shew from the principles of the seventh section, that velocity at B : velocity at C :: CF : CE.

8. If a body having descended from A to B, be projected with the velocity acquired, in a direction making an angle of 30° with the distance: shew that the centripetal force at B : centrifugal force at B :: $2AS$: AB.

9. Suppose a body to descend from rest at A in the right line AS acted on by an attractive force which $\propto \frac{1}{D^2}$ from S, and a repulsive force which $\propto \frac{1}{D^3}$ from S, and let the attractive force be to the repulsive force, at first, as n : 1; find by means of prop. 39, where the velocity is greatest, and where the body will cease to descend.

10. A body moving in a parabola (force in focus) arrives at the extremity of the latus rectum ($4a$). Shew that if f : $2m$ :: force at the vertex : force of gravity, and a velocity $= \sqrt{\frac{fa}{2}}$ be communicated to the body in the direction of the latus rectum, it will describe a circle round the focus. Compare also the periodic time in this circle with the time of moving from the vertex to the latus rectum.

11. Four equal bodies ($F \propto D$) are fixed in the corners of a square. Shew that a body projected in the direction of one of the sides, from the middle point, with twice the velocity acquired in falling from rest to one of the bodies, through a space $= \frac{1}{2}$ the side of the square, will describe the inscribed circle.

12. If P be a particle situated in a homogeneous solid, and a surface be described through P similar to the outer surface, P will not

be affected by the attraction of the matter between the two surfaces. $\left(\text{Attraction varying as } \frac{1}{\text{dist.}^2}\right)$.

13. Resistance varies partly as the velocity, and partly as the velocity squared. Construct for the time when a body descends in such a medium acted on by gravity, first, from rest, secondly, when projected with a velocity greater than any acquirable from rest (Book II. Prop. 13.)

14. Find the horary variation of inclination of the lunar orbit to the plane of the ecliptic (Book III. Prop. 34.).

St. JOHN'S COLLEGE, May 24, 1823.

1. Explain what is meant by angular velocity, and how it is measured; and determine those points in an ellipse, where the angular velocity round the focus is a mean proportional between the greatest and least.

2. In two equal circles, absolute forces, which are as 4 : 1, are situated in the centre of one, and in a point within the other which bisects the radius. Required the ratio of the periodic times.

3. Find the actual periodic time in a given ellipse, and the velocity at any assigned point; the centre of force being in the focus.

4. In the hyperbolic spiral, compare the centripetal and centrifugal focus at any point; and the area *dato tempore* with the area described in the same time in a circle at the same distance.

5. If (l) be the latus rectum of a parabola, (r) the radius vector, and $\pi = 3{\cdot}14159$ &c., the time of descending from rest from any point to the focus : the time of revolving in the curve from that point to the vertex :: $3\pi r^{\frac{3}{2}} : (2r + l) . \sqrt{(4r - l)}$. Required a proof.

6. A body revolves in a circle, in whose centre is an attractive force varying as $\frac{1}{D^3}$. The absolute force being suddenly doubled, what change will be produced in the body's orbit; and through what angle will it revolve before it falls into the centre.

7. In Prop. 44, suppose the immoveable orbit to be a circle having the centre of force in its circumference; and let $G : F :: 3 : 1$. Find the equation to the orbit in fixed space, and the law of the force by which a body may be made to revolve in it.

8. Supposing *P*'s orbit to be originally eccentric, explain clearly the motion of its apsides occasioned by the disturbing force of the body *S*, during one synodic revolution. (Prop. 66. Cor. 8.)

9. Find the force with which a corpuscle, placed in the centre of a sphere, is attracted towards any given segment. (Prop. 83.)

10. In any given position of the Moon's nodes, the mean horary motion in one revolution equals half the greatest horary motion when the Moon is in syzygy. (Book iii. Prop. 30. Cor. 2.)

St. JOHN'S COLLEGE, 1824.

1. Explain what is meant by the terms 'limit' and 'limiting ratio' of evanescent quantities. And from your definition answer the objection which Newton mentions, viz. that there can be no such ratio, because the ratio which the quantities bear to each other before they vanish is not their ultimate ratio, and that when they have vanished they have no ratio at all. Find also what is the limiting ratio of the excess of the tangent of a circular arc above its chord to the excess of the chord above the sine, when the arc vanishes.

2. AT touches a circular arc AB at the point A, AM is perpendicular to AT, arc $AB = AT$, join TB, produce it to meet AM in M, find the limit of AM when the arc vanishes.

3. If AP be a part of a curve and if $AN = x$, $NP = y$: and $y = ax^\alpha + bx^\beta +$ &c. where α, β, γ, &c. are taken in order, beginning with the least. Shew that if α lies between 1 and 2, no circle however small can be drawn touching AP in A so that the arc AP shall lie entirely without it: but that if α lies between 2 and 3, no circle however great can be drawn so that the arc AP shall be entirely within it.

4. Having given P the periodic time of one of Jupiter's satellites, (a) the semi axis of the ellipse it describes, and R the radius of the

planet itself, find from thence the length of a pendulum which would oscillate seconds at Jupiter's surface.

5. A body revolves in an hyperbola, by a force which always acts in the direction of its ordinate, shew that the force varies inversely as (ordinate)^3.

6. If a body revolves in a circle acted upon by a force tending to a point in its circumference, shew from the expression

$$F \propto \lim. \frac{QR}{SP^2 . QT^2},$$

that the force varies inversely (dist.)^5, find also, having given the intensity of the force, the velocity of the body at any point, and prove that this velocity is equal to that which is acquired in falling from infinity.

7. If force $\propto \frac{1}{D^n}$, n being greater than 3, shew that if a body be projected from an apse with the velocity acquired in falling from infinity, the number of revolutions it would describe before reaching the centre $= \frac{1}{2(n-3)}$. And shew how you apply this expression to the case where $n = 5$ and where it appears from the preceding question that the body describes a semi-circle in descending from an apse to the centre of force.

8. An imperfectly elastic body falls from an infinite distance towards a centre of force, whose intensity $\propto \frac{1}{(\text{dist.})^2}$, and impinges upon a perfectly hard plane inclined at an angle of 45° to the direction in which it is falling, shew that the axis major of the ellipse described $= \frac{2r}{1 - m^2}$, and latus rectum $= r(1 + m)^2$; r being the distance of the impinging body from the centre of force, and m the force of elasticity, and shew how the position of the axis-major may be calculated.

9. If a body revolve in an orbit round a centre of force, and if the force at any distance $(r) = P$ and $u = \frac{1}{r}$; and $\frac{h}{2} =$ area described

in $1^{''}$ and v the angle described measuring from any given radius vector. Shew that $\frac{d^2u}{dv^2} + u - \frac{P}{h^2u^2} = 0$; and from this equation prove that the difference of the forces by which one body revolves in a fixed orbit, and another in a revolving orbit, as in Section 9 of Newton, $\propto \frac{1}{D^3}$.

10. Find the horary variation of inclination of the lunar orbit to the plane of the ecliptic.

11. If the force $\propto$ distance, two spheres will attract each other with forces varying as the distance between their centres.

St. JOHN'S COLLEGE, 1825.

1. Explain the method of limiting ratios, and find the limiting ratio of an hyperboloid to its circumscribing cylinder, when indefinitely great and indefinitely small.

2. Prove Lemma 9, and shew why AD, DB, must both make finite angles with the tangent.

3. Find the force tending to the centre of a given ellipse, and investigate the method.

4. Force varying in a lower law that $\frac{1}{D^3}$; shew that a body projected from an apse, with a velocity less than the velocity in a circle at the same distance, must come to a second apse.

5. Compare the times of falling into the centre from a given distance by two forces, one varying as the distance, the other $\frac{1}{D^2}$; the forces at first being supposed equal.

6. Find where a body in the ellipse approaches the centre fastest.

7. When a body falls from A to B by an attractive force, and another rises from B to A by the same force now supposed repulsive, are the velocities acquired and times of motion equal in the two cases?

8. Circular motion is sustained by a crank impelled by an uniform force which acts in both directions and always passes through a fixed point. Shew that the effect is the same whatever be the distance of that point.

9. In the vertex of an equilateral cone is placed a force $\propto \frac{1}{D^5}$. Find the curve described on its surface by a body projected from an apse with velocity from infinity.

10. If S and P mutually attract each other, force $\propto \frac{1}{D^2}$, the axis major of the ellipse described by P round S : axis major of ellipse described by P round S fixed in the same time :: $\sqrt[3]{(S+P)} : \sqrt[3]{S}$. Prove this, and apply it to compare the quantities of matter in S and P.

11. A weight is suspended by a rope reaching from the Earth's surface half way to the centre. Required the variation of the rope's thickness that it may be equally strong throughout.

12. Shew how the actual quantity of matter and mean density of the Earth may be determined.

13. Find the attraction of a straight line to another bisecting it at right angles, and account for the result.

14. Give the method of comparing the diameters of the Earth. (*Newton* Prop. 19. Vol. III.)

15. Prove that the mean decrement of the motion of the Moon's nodes to be subtracted from their mean motion $= \frac{1}{4}$ of the decrement in syzygy. (*Newton* Cor. Prop. 31. Vol. III.)

ST. JOHN'S COLLEGE, DEC. 1827.

Sections I, II, III.

1. EXPLAIN what is meant by continued finite curvature, and prove that if the subtense of the angle of contact do not vary ultimately as the square of the conterminous arc, the curvature is not finite. Is the curvature finite at a point of contrary flexure.

2. Find the eccentricity of an ellipse, when the time from the perihelion to mean distance : time from mean distance to aphelion :: 1 : n.

3. Required the law of force by which a body may describe an hyperbola, the force acting in lines perpendicular to the axis minor.

4. A body is projected at an angle of 30° with a velocity which is equal to the velocity in a circle whose radius is $\frac{2}{3}$ of the distance.—Required the diameters of the orbit described, its eccentricity, and position of the apse $\left(F \propto \frac{1}{D^2}\right)$.

5. Find the space due to the velocity at any point of a curve by the action of the force at that point continued uniform.

6. A body describes a parabola round a centre of force in the vertex, find at what point in the orbit its velocity is equal to the velocity in a circle at the same distance.

7. A string bearing a weight P at its extremity is just strong enough to support it after oscillating through 60°. Shew that the angle θ, through which it may oscillate so as just to support a weight Q may be determined from the equation $\theta = \text{vers.}^{-1}\left(\frac{P}{Q} - \frac{1}{2}\right)$.

8. Determine generally the curve which is the locus of the centre of force, so that a body may describe any orbit with an uniform velocity; and give an example in the case of the logarithmic spiral.

9. The Earth being supposed spherical, a perfectly elastic ball is projected in a direction making an angle of 45° with the horizon, and with a velocity = the velocity in the circle at the same distance. Shew that after 3 rebounds it will return to the same point, and prove that the whole time of motion : periodic time in a circle at the Earth's surface :: $2(\pi + \sqrt{2}) : \pi$.

10. In the above problem find the position of a circular ring so that the ball may just touch it, and supposing a body to fall from this ring, determine the whole time of descent to the earth's centre.

11. A body is projected in the plane of the Earth's orbit at right angles to the distance from the Sun, from a point which equals half the Earth's distance, and with a velocity which is double the Earth's velocity. Shew that the time it remains within the Earth's orbit : the length of a year :: 1 : 3π.

12. A right angled cone is suspended at its vertex A, and its side AB is kept vertical by a ring B. With what angular motion must the cone revolve round AB, in order that there may be no pressure at B.

St. JOHN'S COLLEGE, Dec. 15, 1828.

Sections I, II, III.

1. Prove the 2nd Lemma; and shew that if the bases of the inscribed rectangles be bisected, and if rectangles be formed on these bases as before; that the difference between the curvilinear area and the sum of these inscribed rectangles is ultimately half what it was in the former case.

2. Prove the 11th Lemma, and determine whether the point I moves to or from A in a curve whose equation is $x = ay^2 + by^n$.

3. Is the line joining the bisection of any arc of a curve and the middle point of its chord ultimately a normal to the curve?

4. If a body describe a circle round two centres of force R and S, in the same time; prove that at any point P, the velocity round S : velocity round R :: RP : SG; SG being parallel to RP, and hence construct for the point where the velocities are in a given ratio.

5. A body cannot describe a curve uniformly except by the action of forces in the direction of the normal and varying inversely as the radius of curvature at each point.

6. Compare the force at any point of the logarithmic spiral, with the force in a circle described at the same distance and with the same angular velocity.

7. If a body be projected, at a given distance and with a given velocity, round a centre of force which varies inversely as the square of the distance; find the greatest and least possible eccentricities of the described orbit.

8. In different conic sections round the same centre of force in the focus, the area described in a given time varies as the square root of the latus rectum.

9. Given the radius of the Earth, the velocity of a point in the equator and the space through which a body falls in 1″ at the equator; find the number of days in which a satellite would revolve round the Earth at its surface.

10. With the data of the last problem, find the orbit which a body will describe, projected vertically from a point in the equator, with a velocity equal to the velocity of the point of projection.

11. Prove that a body attracted to two centres of force A and B may describe an infinite number of circles in planes perpendicular to AB, and find the form of the surface of revolution which they all form, and the velocity in any given circle, when the forces both vary as $\frac{1}{d^2}$.

12. If a given heavy rod revolve in a vertical plane, with such a velocity as not to press on the fulcrum when in its highest position, find the tension of any point of the rod in any position.

ST. JOHN'S COLLEGE, MAY 1830.

1. ENUNCIATE and prove Lemmas 10 and 11.

2. The limiting ratio of the sagittæ which bisect the chords and converge to a given point is the same with that of the squares of the arcs, chords, and tangents.

3. If bodies describe similar figures round centres of force similarly situated, the forces at similar points $\propto$ (distance) $\div$ (time of describing similar arcs)^2.

4. In the same or different orbits, if the sagittæ of arcs be drawn through the centres of force, the centripetal forces at the middle of those arcs are in the limiting ratio of the sagittæ directly and squares of the times inversely.

5. Find the law of force tending to the pole by which a body may describe the equiangular spiral.

6. Find the law of force tending to the focus by which a body may describe a parabola.

7. Find the periodic time in an ellipse round the focus.

8. Determine the axes and eccentricity of the conic section described round a centre of force $\propto$ (distance)$^{-2}$. V being the velocity, $\frac{m}{D^2}$ the force, and ϕ the angle between the direction of the motion and the radius vector, at the distance D.

9. Centrifugal force $= \frac{h^2}{SP^3}$.

10. When a body descends in a straight line towards a centre of force $\propto$ (distance)$^{-2}$, compare the velocity at any point with the velocity in a circle at the same distance.

11. Determine the time of describing a given space when a body is projected from a given point with a given velocity, towards or from a centre of force $\propto$ (distance)$^{-2}$.

12. Find the internal space due to the velocity in an ellipse described round a centre of force $\propto$ distance.

St. JOHN'S COLLEGE, Dec. 1830.

Sections I, II, III, VII.

1. Prove Lemma 7. If AED be the tangent at A, as in the Lemma, shew how ED varies in terms of AE or AD; and thence explain clearly by the principles of Lemma 1, that the limiting ratio of $AE : AD$ is a ratio of equality.

2. ABC is a right-angled triangle, whose hypothenuse remains constant; prove that, if AB, ab be two positions of the hypothenuse intersecting in P, triangle PAa : triangle PBb :: CB^4 : CA^4 ultimately.

3. Find the law of force in the orbit, where the angular velocity varies inversely as the perpendicular from the centre of force upon the tangent.

4. If two equal centres of force, which varies as the distance, be placed in the extremities of the diameter of a circle, a body acted on by these forces may describe the circle uniformly; determine also the velocity.

5. Deduce from the expression, $F = \frac{2h^2}{SP^2} \cdot \text{limit} \frac{QR}{QT^2}$ the law of force in a parabola, when the force acts parallel to its axis.

6. In an ellipse the angular velocity of CP, force in focus, : its angular velocity, force in centre, :: P's linear velocity in the former case : its linear velocity in the latter.

7. Bodies of different elasticities, describing a circle round a centre of force in the centre which varies as $(\text{dist.})^{-2}$, strike one of the radii produced; determine the curve, in which lie the extremities of the minor axes of the new orbits.

8. The diminution of gravity, arising from centrifugal force, at a place on the Earth's surface, whose angular distance from the pole, measured at the centre, is 45°, is $\left(\frac{1}{n}\right)^{\text{th}}$ of that at the equator, shew that the Earth's eccentricity $= \left(\frac{4 - n^2}{8 - 3n^2}\right)^{\frac{1}{2}}$ nearly.

9. If Px, taken in PS, be the space due internally to the velocity in an ellipse at P, the locus of x is an ellipse.

10. A body falls from rest towards a centre of force S, which $\propto (\text{dist.})^{-2}$; determine the time of describing a given space. (Prop. 36.)

11. In the last equation, if θ be the angle, which the arc, whose versed sine is the space fallen through, subtends at S, t the time of

motion, and T the time through the same space by the action of the force at the beginning of the motion continued uniform, then $t = T . \cos.^2 \frac{\theta}{2}$.

12. A planet is at its greatest angular distance (α) from the Sun t days after passing between the Earth and the Sun, shew that if the Earth and planet describe circles in the same plane, the planet's periodic time $= \frac{4\pi t}{\pi - 2\alpha} \cdot (1 - \sin.^{\frac{2}{3}} \alpha)$.

St. JOHN'S COLLEGE, Dec. 1830.

Sections IX, XI.

1. Why cannot the principles of the ninth Section be applied to determine the motion of the lunar apsides? Shew clearly, supposing the objection did not exist, how the application might be made.

2. The difference of the forces, by one of which the body would be retained in the fixed, and by the other in the moveable orbit, varies as $(\text{dist})^{-3}$.

3. When P's orbit is a straight line, and is projected *in consequentiâ*, construct for the orbit described by p.

4. In orbits nearly circular, having given the law of force, find the angle between the apsides.

5. The angle between the apsides $= 360 \sqrt{\frac{b + c}{mb + nc}}$; determine the law of force.

6. The curves described by two bodies S and P, attracting each other, round the centre of gravity, are similar, and the curve, which P appears to a spectator at S to describe, is similar to either of them.

7. Two bodies, revolving round their common centre of gravity, describe round it areas proportional to the times.

8. Prove Prop. 66.

9. Define "addititious," "ablatitious", "central disturbing force:" deduce an expression for the last mentioned, and shew when it equals 0.

10. Explain clearly, why the lunar months are longer in winter than in summer.

11. Determine the effect of the disturbing force on the motion of the Moon's nodes.

12. If the system of P and T remain the same, whilst S and ST vary, determine the variations of the angular errors of P as seen from T, caused by the disturbing force of S.

St. JOHN'S COLLEGE, Dec. 1831.

Sections I, II, III, VII.

1. Enunciate and prove Lemma 4. Shew that it is true for quantities of any kind; and thence apply it to prove that a cone $= \frac{1}{3}$ of its circumscribing cylinder.

2. The area of the conchoid of Nicomedes, between the vertex A and an ordinate PN to the axis : the inscribed triangle APN :: 4 : 3 ultimately.

3. Find that point in the diameter of a given circle round which as a centre of force a body may revolve in the circle, so that the angular velocities at the apsidal distances may be in a given ratio.

4. Supposing a comet in its perihelion to be suddenly acted upon by the Earth, which is also in its perihelion, with a force $= \frac{1}{n}$th of that of the Sun, shew what change will be produced in the Comet's orbit; the perihelion distances and the planes of the two orbits being supposed to coincide.

5. Find the nature of the orbit, in which the difference between the centripetal and centrifugal forces always $\propto \frac{1}{(\text{dist.})^5}$.

6. Find the space due to the velocity at any point of any curve, by the action of the force at that point continued uniform; and thence determine the velocity in an ellipse at the mean distance, the force varying as $\frac{1}{(\text{dist.})^2}$.

7. A body moving in an ellipse round a centre of force situated in one of its foci, is reflected at the extremity of the latus rectum, in a direction tending directly to the other focus, so as to lose no velocity: shew that the periodic times in the new orbit and original ellipse are equal, and determine the inclination of their axes to each other.

8. Supposing the time curve in Prop. 39 to be a rectangular hyperbola between its asymptotes, find the equation to the velocity curve. Determine also the law of force, and the position of the centre.

9. The force varying as the distance, a body descends from a given point A towards a centre of force S, and is acted upon at the same time by a constant force in directions always perpendicular to AS. Determine the motion of the body.

10. If (r) be the radius vector belonging to any point of any curve described round a centre of force which varies as $\frac{1}{(\text{dist.})^2}$; and δ, δ' be the external and internal spaces due to the velocity at that point respectively; prove that the chord of curvature, at that point, drawn through the centre of force $= 4r\left\{\frac{\delta - \delta'}{\delta + \delta'}\right\}$.

11. A body A connected by an elastic string with a fixed point S, is acted upon by a constant force in the direction SA. Having given the extensibility of the string, find the whole distance through which the body will be drawn, and the time of motion.

12. If the eccentricity of a planet's orbit $= \frac{1}{2}$, shew that the time of moving from the perihelion through an angle of $90° = \frac{1}{6}$ of the periodic time, diminished by $\frac{1}{2}$ of the time in which the planet would descend from the aphelion to the Sun, with the velocity at the perihelion continued uniform.

ST. JOHN'S COLLEGE, DEC. 1831.

Sections IX, XI.

1. THE body will be at an apse in the orbit *Vpk* at the same time as in the orbit *VPK*. (Cor. 1. Prop. 43.)

2. In orbits nearly circular, having given the law of force, find the angle between the apsides; and apply the method to the case in which the force is constant.

3. If *S* and *P* attracting each other describe curves round a common centre of gravity *C*, the curves will be similar; and the apparent orbit of *P* to a spectator at *S* will be similar to either of the curves round *C*.

4. Enunciate and prove Prop. 66.

5. Find the value of the mean central disturbing force; and determine that part of the ablatitious force, which acts perpendicularly to the plane of *P*'s orbit.

6. Explain the effect of the disturbing force on the motion of the apsides, in an orbit nearly circular, during one revolution of *P*.

7. In different systems, having given only the form and inclination of the orbits, to compare the periodic linear and angular errors, and also the mean angular errors as seen from *T*.

ST. JOHN'S COLLEGE, MAY 1831.

Sections I, II, III, VII.

1. DEFINE a limiting ratio, and from the definition find the limiting ratio of the surface of a paraboloid to that of its inscribed cone.

2. Prove that cycloids are similar figures; and spirals, the equations of which are of the form $r = a \sec.^2 \frac{\theta}{2}$.

3. Enunciate and prove Lemmas 2 and 9.

4. The limiting ratio of the sagittæ, which bisect the chords of two conterminous arcs and converge to a given point, is the same with that of the squares of the conterminous arcs, chords and tangents. (Cor. 2. Lemma 11.)

5. If a body moving in a curve described areas proportional to the times by lines drawn from the body to any the same point, the body is retained in the curve by a force tending to that point. (Prop. 2.)

6. In any places whatever, having given the velocity with which a body describes a given figure by forces tending to some common centre, to find that centre. (Prop. 5.)

7. Find the law of force tending to the centre by which a body may describe an ellipse. (Prop. 10.)

8. If a body be projected from a point P, in a direction zPy, which makes an angle with the distance SP, and be urged by a force tending to S, which varies as $\frac{1}{(\text{dist.})^2}$, it will describe a conic section of which S is the focus. (Cor. 2., Prop. 13.)

9. In the preceding question, having given the force, and velocity of projection, determine the axes and eccentricity of the orbit described.

10. Find the periodic time in an ellipse, the centre of force being in its focus.

11. Compare the angular velocity of the distance SP, in any orbit, with that of SY the perpendicular upon the tangent.

12. Prove that the centripetal and centrifugal forces at any point of a curve are respectively $\frac{h^2}{p^3} \times \frac{dp}{dr}$, and $\frac{h^2}{r^3}$; (where (h) is twice the area described in $1''$, and r and p are respectively the radius vector and perpendicular upon the tangent.) Apply the former expression to find the law of force by which a body may describe the equiangular or logarithmic spiral.

13. The force varying as $\frac{1}{(\text{dist.})^2}$, and the body descending in a line which is the limit of an ellipse or hyperbola, the velocity at any point C : velocity in a circle (radius BC) :: $\sqrt{(AC)}$: $\sqrt{(\frac{1}{2}AB)}$. (Prop. 33.)

14. The area DES (Prop. 32.) described by the indefinite radius SD during the motion of the body from C to S, is equal to the area described in the same time by a body revolving uniformly in a circle, the radius of which is $\frac{1}{2}$ latus rectum of DES. (Prop. 35.)

15. Find the external and internal spaces due to the velocity in an ellipse, when the force varies as the distance.

16. The force varying according to any given law, and the body ascending or descending from rest, to find the velocity it has acquired at any place, and the time of its motion. (Prop. 39.)

St. JOHN'S COLLEGE, May 1831.

1. Find an expression for the force by which a body may be made to describe any orbit round a fixed centre in the same plane; and apply it to find the law of force tending to the focus of an ellipse.

2. A body is projected from an apse, with a velocity equal to twice the velocity in a circle at the same distance, and is attracted by forces tending to the same centre, one varying directly as the distance, the other inversely as its square; if at the point of projection the latter force be double of the former, find the equation to the orbit described.

3. In the above problem, if a and v denote the initial distance and velocity, shew that the body will come to a second apse, at a distance $= 3a$, after describing an angle

$$= \pi\left(1 - \frac{1}{\sqrt{5}}\right), \text{ in a time} = \frac{2\pi a}{v}\left(\sqrt{3} - \frac{2}{\sqrt{5}}\right).$$

4. If the axes of an homogeneous oblate spheroid be to one another as $\sqrt{5} : 1$, its moment of inertia is the same about all axes passing through one of its poles.

5. If a body be projected along the concave superficies of a surface of revolution, not in a plane through the axis, and be acted upon by no force; prove that its path will constantly cut the generating curve at an angle whose sine varies inversely as the distance from the axis.

6. The mean horary variation of the inclination of the lunar orbit to the plane of the ecliptic, for a given position of the line of nodes, varies nearly as the sine of twice the angular distance of the nodes from syzygy. (Lib. 3. Prop. 34.)

7. In the Lunar Theory, given that

$$d_\theta^2 s + s + \frac{3}{2}m^2 k\,[\sin.(g\theta - \gamma) - \sin.\{(2 - 2m - g)\theta + 2\beta + \gamma\}] = 0,$$

find s, and explain the effect of the terms in the expression.

8. If a comet describe an hyperbola (major axis $= 2a$) about the Sun, and the length of the line of nodes $= a(\sec.\theta - 1)$, prove that the time of passage between the nodes through the perihelion

$$= \frac{Pa^{\frac{3}{2}}}{2\pi}\{\tan.\theta - \log.(\sec.\theta + \tan.\theta)\},$$

P being the Earth's period at the mean distance 1.

9. The sides of an isosceles triangle are formed of slender uniform prisms, attracting with forces $\propto D^{-2}$; find the vertical angle, in order that a particle may remain at rest in a point, which divides the perpendicular from the vertex in a given ratio.

10. Let a, b, c, a', b', c', be the $\frac{1}{2}$ axes of two ellipsoids whose principal sections are in the same planes, and have the same foci; P' a point in the surface of the first, and P a point in the surface of the second, so situated that their co-ordinates are proportional to the $\frac{1}{2}$ axes to which they are parallel; A, B, C, A', B', C', the forces parallel to the axes, with which the first attracts P (a point exterior to it), and the second P' (a point within it); prove that

$$Ab'c' = A'bc, \quad Ba'c' = B'ac, \quad Ca'b' = C'ab.$$

11. State and prove the principle of the conservation of areas.

12. A body, repelled from a centre of force varying directly as the distance, is constrained to move uniformly by the resistance of the medium; find the nature of its path.

13. From what series of points, in a plane passing through two centres of force, may a body be projected in a direction perpendicular to the plane, so as to describe a circle? and with what velocity must it be projected from any given one of them? force $\propto D^{-2}$.

14. A solid of revolution moveable about its centre of gravity G, which is the origin and fixed, and having its axis inclined to the axis of z at an angle ϕ, has an angular motion impressed upon it about a line between these two axes, and inclined to the former at an angle θ, such that $k^2 \tan.\phi = k_1^2 \tan.\theta$, (where k, k_1 are the radii of gyration of the body about its axis, and a line perpendicular to the axis through G); prove that the axis of the solid will constantly preserve the same inclination to the axis of z, and will revolve uniformly about it; and the solid will at the same time revolve uniformly about its own axis, which is in motion.

15. Required the part of the Sun's disturbing force perpendicular to the plane of the Moon's orbit.

16. For a homogeneous revolving fluid mass, the oblate spheroid is the form of equilibrium, if the force at the pole : whole force at the equator = equatorial axis : polar axis.

St. JOHN'S COLLEGE, May 1832.

Sections I, II, III, VII.

1. Enunciate and prove Lemmas 1, 6.

2. Enunciate and prove Lemma 10. Are the quantities treated of in this Lemma represented by quantities of the same kind?

3. If a body moves in a curve line in a constant plane, and by a ray drawn to a fixed point describes areas about that point proportional to the times, it is urged by a central force tending to that point.

If the description of areas be retarded, the direction of the force will be opposite to that in which the body is moving.

4. Compare the forces at two similarly situated points in similar curves having centres of force similarly situated.

5. Find the space due to the velocity in a circle, the body being urged by a force equal to the force in the circumference continued uniform.

6. In the same or different orbits, if the sagittæ of arcs be drawn through the centres of force, the centripetal forces at the middle points of the arcs $\propto$ limit (sagitta) $\div$ (time of describing arc)^2.

7. A body revolves in the circumference of a circle, the force tending to any point: find the force.

8. Having given, at a given point, the velocity and direction of the motion of a body, revolving about a given centre of force varying as the distance, find the curve described.

9. A body moves in a hyperbola: find the force tending to the focus.

10. A body revolves in an ellipse: find the periodic time.

11. Find the angular velocity of a body revolving in a curve, in terms of its distance from the centre of force, and of the area described in the time 1″.

12. A body descends from rest urged by a force varying as the distance: find the velocity of the body at any point, and the time of describing a given space.

13. Find the velocity at any point, and the time of describing a given space, when the force varies inversely as the square of the distance.

CAIUS COLLEGE, MAY 1831.

1. ENUNCIATE and prove the ninth Lemma.

2. A body being acted on by a central force, the areas described are in one plane, and proportional to the times of description.

3. A body revolves in an ellipse: required the law of force tending to the centre.

4. A body is projected with a given velocity, in a given direction, and at a given distance from a fixed centre towards which it tends;

$F \propto \frac{1}{D^2}$; determine the form and dimensions of the orbit described.

5. The velocity at any point in a curve is equal to that which would be generated by the force at that point, continued constant, and acting on a body while it moves through one fourth of the chord of curvature at that point drawn through the centre of force.

6. A body falls from rest from a given point towards a centre of force, $F \propto \frac{1}{D^n}$: required the values of n when the time of describing any space can be found.

$F \propto D$, find the time to the centre.

7. The difference of forces by which a body may be made to revolve in the fixed and moveable orbits $\propto \frac{1}{(\text{Dist.})^3}$.

8. Find the time of an oscillation in the epicycloid, the force tending to the centre of the globe, and varying as the distance.

9. If any number of bodies be acted on only by their mutual attraction, their centre of gravity will either be at rest or will move uniformly forward in a straight line.

10. The inclination of the Moon's orbit is greatest when the line of nodes is in syzygies, and least when it is in quadratures.

11. Find the attraction of a sphere on a particle without it, the attraction of each particle varying as the distance.

12. Find the horary increment of the area which the Moon describes about the Earth in a circular orbit.

CAIUS COLLEGE, June 1832.

1. Explain fully what is meant by the expressions "ultimate value" and "ultimate ratio;" and what is the general method of reasoning in Newton's Lemmas.

2. Prove Newton's ninth Lemma. When is the curvature of a curve said to be continuous?

3. A body, acted on by a force tending to a centre, will describe, by radii drawn to that centre, areas in one plane and proportional to the times.

4. Prove Newton's expression for the law of force by which a body will describe a given curve; and find by what law of force tending to the centre, a body will describe an ellipse.

5. A body is projected from a given point, in a given direction, with a given velocity, and is acted on by a force tending to a centre and varying as $\frac{1}{(\text{dist.})^2}$; find the orbit described.

6. On the same supposition, shew that the major axis of the orbit is independent of the direction of projection, and that the form of the orbit depends solely on the direction of projection and the ratio of the velocity of projection to the velocity in a circle at the same distance.

7. State fully the several steps by which Newton proceeds, in the ninth section, to find the angle between the apsides in orbits nearly circular. Why does this method fail of determining the amount of the progression of the Moon's apogee?

8. A body moves on a surface of revolution and is acted on by a force tending to a centre situated in the axis. Shew that the projection of the radius-vector joining the centre and the body, upon a plane perpendicular to the axis, will describe areas proportional to the times.

9. Prove Prop. 66. Sect. 11.

10. Shew that by the disturbing force of the Sun,

(1). The distance from the Earth and periodic time of the Moon, are increased above what they would be in the undisturbed orbit.

(2). An inequality is produced in the description of areas, whose period equals half the synodic period of the Sun and Moon.

11. Find the horary motion of the Moon's nodes in a circular orbit. (*Newton*, Vol. 3. Prop. 30).

12. The Moon's longitude being expressed (nearly) by

$$pt + 2e \cdot \sin.(cpt - \alpha) + \frac{5e^2}{4} \cdot \sin.(2cpt - 2\alpha).$$

$$- \frac{k^2}{4} \cdot \sin.(2gpt - 2\gamma) + \frac{11}{8} m^2 \cdot \sin.\{(2 - 2m)pt + 2\beta\}.$$

$$+ \frac{15}{4} me \sin.\{(2 - 2m - c)pt + 2\beta + \alpha\} - 3me'.(\sin.mpt - \beta - \zeta).$$

Explain what is indicated by the several terms.

13. The moving force by which two homogeneous spheres attract each other is as the product of the masses directly, and the square of the distance of their centres inversely.

14. Investigate the forces exerted by one planet to disturb the orbit of another, and the differential equations of their motions.

JESUS COLLEGE, June 1832.

Sections I, II, III.

1. If in two curvilinear figures there can be inscribed the same number of parallelograms which, when their number is increased sine limite, are ultimately to each other in a given ratio, the areas of the curvilinear figures are in that ratio.

2. The chord, arc, and tangent in a curve of continued curvature are ultimately ratios of equality.

3. The spaces described from rest by a body acted on by any finite force, are in the beginning of the motion as the squares of the times.

4. The limit to which every curve of finite curvature approaches is the common parabola.

5. If a body moving in a plane curve describes areas proportional to the times, by lines drawn from the body to any point, the body is retained in the curve by a force tending to that point.

6. If AB, BC be the spaces described in two equal successive portions of time, the force being supposed to act by impulses at A and B, prove that the tangent at B to the actual path of the body is ulti-

mately parallel to AC; and thence shew that the space described by the action of the force = half the space described in consequence of the impulse in the same time.

7. Find the space due to the velocity at any point of a curve by the action of the force at that point continued uniform.]

8. Prove that force $= 8a^2 . \text{limit} . \frac{QR}{SP^2 . QT^2}$.

If the force act in parallel lines, parallel to the axis of a parabola, apply the above expression to find the law of force.

9. If a body be projected in a direction making any angle with the distance from a fixed point, and be attracted to that point by a force varying as the distance, it will describe an ellipse or circle whose centre is the centre of force.

10. Find the law of force tending to the focus by which a body may describe an ellipse; find also the velocity at any point of the ellipse.

11. The periodic times in different ellipses round the same centre of force $\propto$ (axis major)$^{\frac{3}{2}}$.

12. The angular velocity round the farther focus of an ellipse of small eccentricity is nearly uniform.

SIDNEY SUSSEX COLLEGE, May 1830.

Sections I, II, III.

1. Find the general equation to a conic section, considered as the locus of a point whose distances from a given point and a straight line given in position are in a constant ratio, and particularize the three cases.

2. Define the subtangent and subnormal in a curve, and show that in the parabola, latus rectum : subnormal :: subtangent : abscissa.

3. Show that in all the conic sections, if GK be drawn from the foot of the normal perpendicular to the distance SP, $PK = \frac{1}{2}$ latus rectum.

4. Explain clearly what is meant by the ultimate ratio of two quantities; show that such a ratio exists, and apply the first lemma to evanescent quantities.

5. Enunciate and prove Newton's 9th Lemma, and show why it will not apply to prove the 10th, when the force is infinitely great, or when it is infinitely small.

6. Define finite curvature, and prove the converse of the eleventh lemma.

7. If a body be attracted to a fixed centre, the force at any point of the orbit described $= 2h^2 . \text{limit} \frac{QR}{SP^2 \times QT^2}$, if $h = 2 \times$ area described in 1″. Prove this, and mention briefly how Newton obtains the different propositions assumed in the proof.

8. At similar points in similar curves, described round centres of force similarly situated, prove that the forces $\propto \frac{(\text{vel.})^2}{\text{dist.}} \propto \frac{\text{dist.}}{(\text{per. time})^2}$.

9. If a body be projected, in a direction inclined at any angle to its distance, from a centre of force which varies as $\frac{1}{(\text{dist.})^2}$, it will describe a conic section.

10. The periodic times in different ellipses round the same centres of force in their foci $\propto$ (axis major)$^{\frac{3}{2}}$.

11. Define paracentric, and transverse velocity, and show that they are equal in the equiangular spiral of which the equation is $r = \epsilon^\theta$.

12. Compare the centrifugal and centripetal forces at any point of a conic section, the centre of force being in the focus, and shew that they are the same at the extremity of the latus rectum.

13. A body is revolving in an ellipse about a centre of force in the focus, which is suddenly transferred to the other focus: construct for the position of the new orbit, find the length of the axis major, and show that the eccentricity is unaltered.

SIDNEY SUSSEX COLLEGE, MAY 1830.

1. HAVING given the time in which a body would fall through a given space at the Earth's surface, the radius of the Earth and the Moon's distance, find the periodic time.

2. Find the time of a body's describing any portion of a parabolic orbit.

3. Investigate Newton's construction for the velocity and time when a body moves from rest directly to or from a centre of force varying according to any given law. Also if force $\propto \frac{1}{d^2}$, and the body descend from an infinite distance, trace the curves and draw their asymptotes.

4. If a body be projected, in a direction inclined at any angle to its distance, from a centre of force which $\propto \frac{1}{d^3}$, find the equation to the orbit described; shew that it possesses five species, and point out which of them have apses and asymptotes.

5. Apply Prop. 44. to determine the whole force at p when the fixed orbit is an ellipse and the force in the focus.

6. If a body oscillate in a hypocycloid, the force being as the distance and tending to the centre of the globe,
time of $\frac{1}{2}$ an oscill. : time to centre :: $\sqrt{(SA)} : \sqrt{(SC)}$. (Prop. 52).

7. The force $\propto \frac{1}{d^2}$; shew that the axis major of the ellipse described by P round s in motion : the axis major of that described by P round s at rest in the same periodic time :: $\sqrt[3]{(S+P)} : \sqrt[3]{S}$. (Prop. 60.)

8. Investigate fully the effects produced on the inclination of P's orbit to the ecliptic by the ablatitious force.

9. If all particles of matter attract each other with forces which $\propto \frac{1}{d^2}$, a corpuscle placed within a spherical shell is equally attracted in all directions.

10. If a body move in a plane acted on by a central force, and if this force $= P$ at a distance $\left(\frac{1}{u}\right)$, the equation to the trajectory described is $\frac{d^2u}{d\theta^2} + u - \frac{P}{h^2u^2} = 0$. (Whewell's *Dynamics.*)

11. If a body move through one or more spaces bounded by parallel planes, and be acted upon by a force which is perpendicular to those planes and which is at the same distance from them, the angle of incidence is to the angle of emergence in a given ratio. (Prop. 94.)

12. *S* and *H* are two equal centres of force, *S* attractive and *H* repulsive. If *SH* be bisected in *C* and *BC* drawn perpendicular to it, shew that a body placed any where in *BC* will describe a semi-ellipse about *S*, of which *BC* is the axis minor, and *S*, *H* the foci.

13. The Moon is retained in her orbit by the force of gravity.

St. PETER'S COLLEGE, May 1831.

1. If a body be acted on by a central force, it's velocity at a given distance will be independent of the curve described.

Shew what condition must be satisfied in order that this may be the case, the body being acted on by any forces. Will it be satisfied if a system of bodies move round a common centre of force, and be acted on by their mutual attractions?

2. If a body be acted on by a central force which varies $\frac{1}{(\text{dist.})^3}$; shew that the differential equation to the curve is

$$\frac{d\theta}{dr} = \frac{-ce\sin.\beta}{r\sqrt{\{c^2(1-e^2\sin.^2\beta)-(1-e^2)r^2\}}},$$

c being the distance, and β the angle of projection; the velocity of projection being *e* times that which would be acquired from infinity by the action of the same force.

Find the integral equation to the orbit, when the velocity is greater than that in a circle at equal distance, and the area described less.

3. Explain fully what is meant by the fixed, and moveable orbits, and the orbit traced out in fixed space, in Newton's 9th Section.

Find the expression for the force in the orbit traced out in fixed space.

What is the object of this Section? To what kind of orbits does it apply; and how is the expression, mentioned in the last question, rendered identical with that for the law of force in any such proposed orbit?

4. If a body move in a cycloid on the surface of an inclined plane, find the time of one oscillation when it descends from a given point, taking account of friction.

5. Two bodies attracting each other with forces which $\propto \frac{1}{(\text{dist.})^2}$ being placed together, one of them is projected with a given velocity; determine their motions, and where they will again be together.

6. Explain clearly what is meant by the plane of the Moon's orbit, and how it passes from any position to the consecutive one when it's inclination is varying.

Investigate the motion of it's nodes. (*Newton*, Prop. 66.)

7. What is the evection? In what corollary does Newton point out the cause of it; and in which does he indicate that of the annual equation?

8. State clearly the nature of the elliptical orbit. (*Newton*, Vol. III.)

9. Deduce the expression for the horary motion of the node in a circular orbit, and thence shew that if N be the longitude of the node,

$$N = -\tfrac{3}{4}m^2\theta - \frac{3m^2}{8(1-m)}\sin.(2\theta - 2m\theta) + \frac{3m^2}{8(1+\frac{3}{4}m^2)}\sin.(2\theta - 2N)$$
$$+ \frac{3m}{8(1+\frac{3}{4}m)}\sin.(2m\theta - 2N)$$

where θ is the longitude of the Moon, and m the ratio of the periodic times of the Sun and Moon.

Give the interpretation of the terms in this expression.

10. Enunciate D'Alembert's principle, as applicable to the case of the action of continuous forces, and also to that of impulsive ones.

11. The centres of oscillation and suspension are reciprocal.

What must be the position of the axes about which the body oscillates?

12. If a rotatory motion, and a motion of translation be co-existent in any system of bodies, shew that in the determination of the value of $\Sigma . mv^2$, we may consider them as independent of each other.

St. JOHN'S COLLEGE, June 1832.

1. The orbit which the Sun appears to describe about a planet is an ellipse; prove this and determine the periodic time.

2. Two bodies start at the same time from the farther apse in an ellipse, one to describe the ellipse, the other the circle on the axis major, the farther focus S being the centre of force in both cases; compare the absolute forces in the two orbits, that the periodic times may be equal.

3. If yP and Pz be the external and internal spaces due to the velocity at the point P in an ellipse, force in focus; shew that Sy, Sp, Sz are in harmonical progression.

4. A weight P suspended by a string (c) is drawn $\theta°$ from the vertical by the action of a force placed in the same horizontal plane with the original position of P, and at a distance (a) from it, shew that the tension of the string $= \dfrac{a \text{ cosec.}\theta - c}{a \text{ cotan.}\theta - c} . Pg.$

5. Investigate the equation to the isochronous curve, when the force is constant, and acts in parallel lines.

6. The perpendiculars drawn from a point in a triangle upon the sides are a, b, c, and the angles which the sides subtend at the point are α, β, γ; shew that a particle placed at that point and acted on by the attraction of the sides $\left(\text{which } \propto \dfrac{1}{d^2}\right)$ will be kept at rest,

if $\dfrac{1}{a} : \dfrac{1}{b} : \dfrac{1}{c} = \dfrac{\sin.\frac{1}{2}\alpha}{\alpha} : \dfrac{\sin.\frac{1}{2}\beta}{\beta} : \dfrac{\sin.\frac{1}{2}\gamma}{\gamma}.$

7. In a catenary attracted to a fixed centre, if P be the force of attraction and p the perpendicular on the tangent from the centre at any point, then the force by which the body would revolve in the curve formed by the catenary $\propto \frac{P}{p}$.

8. If x, y be the co-ordinates of the highest point of the curve described by a body acted on by gravity and projected *in vacuo*, at an angle of inclination (α) to the horizon, their decrements, when the body is projected in a rare medium, in which $R = k$. velocity, are $\frac{kx^{\frac{3}{2}}}{\sqrt{(g \cdot \cot.\alpha)}}$ and $\frac{k \cdot (2y)^{\frac{3}{2}}}{3\sqrt{g}}$.

9. A sphere, when acted on separately by three forces, revolves round three diameters inclined at the same angle to each other and with the same angular velocity, determine the angular velocity and the new axis of rotation, when the three forces are applied at the same instant.

10. Construct for the inclination of the lunar orbit to the plane of the ecliptic at a given time. (*Newton*, Vol. III. Prop. 35.)

11. The equation for determining the projection of the Moon's orbit on the ecliptic is

$$d_\theta^2 u + u - \frac{\frac{P}{u^2} - \frac{T}{u^3} \cdot d_\theta u}{h^2 + 2\int_\theta \frac{T}{u^3}} = 0,$$

P and T being the whole forces on the Moon, parallel and perpendicular to the projection on the ecliptic of the Moon's distance from the Earth.

12. A body P is projected with a given velocity ($a\sqrt{\mu}$) in a direction perpendicular to its distance SA from a centre of force S, (which $\propto$ distance) and which itself moves uniformly with velocity (V) in the direction AS produced. Determine the equation to the orbit described and shew that the motions of P and S are parallel when the co-ordinates of P measured from the original position of S are (a) and $\left(\frac{\pi}{2} - 1\right)V$.

PROBLEMS,

AND

MISCELLANEOUS QUESTIONS.

TRINITY COLLEGE, 1820.

1. WHAT sum of money must be laid out in the 3 per cent consols at $63\frac{3}{8}$ per cent, to produce an income of £400 a year?

2. The sine of any angle of a plane triangle has to the opposite side a constant ratio. What is this ratio?

3. Find by the method of continued fractions a series of fractions converging to $\sqrt{19}$.

4. Given the three sides of a plane triangle, find

   (1). Its area,

   (2). The radius of the inscribed circle.

5. Differentiate the following quantities:

   (1). $u = l.\sqrt{\left(\frac{1+\sin.x}{1-\sin.x}\right)}$;

   (2). $u = \cfrac{x^2}{1+\cfrac{x^2}{1+\text{\&c. } ad\ inf.}}$

6. Find a number which being divided by 2, 3, 5 shall leave for remainders 1, 2, 3 respectively.

7. The angles of any plane triangle being A, B, C, prove that to radius (1),

$$4\sin.A.\sin.B.\sin.C = \sin.2A + \sin.2B + \sin.2C.$$

8. (1). Find the locus of the vertices of all the triangles described on the same base, when one of the angles at the base is always double of the other.

   (2). Hence trisect a given angle.

9. Find the radius of curvature at any point of the common cycloid.

10. In any spherical triangle, the arcs of great circles drawn from the three angular points perpendicular to the opposite sides intersect in the same point.

11. Sum the following series:

(1). $\frac{5}{1.2.3.4}+\frac{7}{2.3.4.5}+\frac{9}{3.4.5.6}+\cdots$ to (n) terms.

(2). $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots$ *ad inf.*

(3). $\frac{1}{1.2.3}+\frac{1}{2.3.4}\cdot\frac{1}{2}+\frac{1}{3.4.5}\cdot\frac{1}{2^2}+\cdots$ *ad inf.*

(4). $\text{Sin.}\theta+x\sin.2\theta+x^2\sin.3\theta+\cdots$ *ad inf.*

12. A sphere of given diameter descends in a fluid, from rest, by the action of gravity; find the greatest velocity it can acquire, its specific gravity being (n) times that of the fluid.

13. (1). Of all quadrilateral figures contained by four given right lines the greatest is that which is inscriptible in a circle.

(2). If a, b, c, d be the sides of this quadrilateral, S its semiperimeter, shew that its

$$\text{area}=\sqrt{\{(S-a)(S-b)(S-c)(S-d)\}}.$$

14. Find the centre of gyration of a given sphere.

15. Any two right lines intersect each other in space; having given their separate inclinations to three rectangular co-ordinates passing through the point of intersection: find their inclination to each other.

16. (1). Trace the curve whose equation is $y^2(c-x)=x^3+bx^2$, and find its area when $b=0$.

(2). The equation to a curve is $y^3-axy+x^3=0$; find the value of the ordinate when a maximum, and the corresponding value of the abscissa. Shew also that it is a maximum and not a minimum.

17. State the principle of virtual velocities; and hence shew that if any system in equilibrium, acted on by gravity alone, have an indefinitely small motion communicated to its parts, its centre of gravity will neither ascend nor descend.

18. Integrate

$$(1).\ \frac{dx}{\sqrt{(A + Bx + Cx^2)}} \qquad (2).\ \frac{dx}{1 + x^3} \qquad (3).\ \frac{d\theta}{(\cos.\theta)^4}$$

and find the relation of (x) to (y) in the equations

$$(1).\ xdy - ydx = ydx \log.\frac{y}{x}.$$

$$(2).\ dx + x^3dx = dy + ydx.$$

19. If two weights acting upon a wheel and axle put the machine in motion, find the pressure upon the axis without taking into account the machine's inertia.

20. If (a) and (b) denote the semi-axes of an ellipse, (θ) the angle at which the radius of curvature (r) at any point cuts the axis, prove that $r = \dfrac{a^2b^2}{(a^2\cos.^2\theta + b^2\sin.^2\theta)^{\frac{3}{2}}}$.

21. The roots of the equation $x^n - px^{n-1} + qx^{n-2} - \&c. = 0$, being α, β, γ, &c. find the value of $\alpha^m + \beta^m + \gamma^m + \cdots$ in terms of the coefficients p, q, r, &c.

22. AP is any arc of a parabola whose vertex is A and focus S; let N be the intersection of a perpendicular from S on the tangent at P with the perpendicular to the axis from A.

Then if $AS = a$, $\angle ASN = \phi$.

Shew that arc $AP - PN = a.\text{l.}\tan.\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$.

23. If a circle whose diameter is equal to the whole tide in any given latitude be placed vertically, and so as to have the lower extremity of its diameter coincident with the level of low water, prove that the tide will rise or fall over equal arcs in equal times.

TRINITY COLLEGE, 1821.

1. A PERSON places £10,000 in the Funds at 5 per cent.: the first year he spends the whole interest, the second, third, &c. years he spends twice, three times, &c. the same interest: how long will his property last?

2. (1). Given $x + y = a$

$$x^3 + y^3 = b^3; \text{ find } (x) \text{ and } (y).$$

(2). Required all the integral values of (x) and (y) in the equation $13x + 14y = 200$.

(3). Find the number of impossible roots in the equation

$$x^4 - 6x^2 - 3x - 2 = 0.$$

3. Reduce an observed oblique angle to the corresponding angle on the horizontal plane.

4. If p, p_1, p_2, denote the projections of any plane figure P on three rectangular co-ordinate planes, prove that

$$P^2 = p^2 + p_1^2 + p_2^2.$$

5. Find the algebraic, and thence deduce the polar, equation to the ellipse: the centre being the origin of the co-ordinates.

6. (1). Investigate by the geometrical analysis the conditions required in finding two mean proportionals between two given straight lines: and

(2). Shew how these conditions may be fulfilled.

7. The equation to lines of the second order being

$$y^2 = mx + nx^2,$$

prove that the radius of curvature varies as the cube of the normal.

8. Find a point in the circumference of a given circle such that the sum of its distances from two given points without it may be a minimum.

9. Given the side of a regular octahedron, find

(1). The inclination of any two adjacent faces.

(2). The radius of the inscribed and circumscribed sphere.

10. Trace the curve whose equation is $x^3 + bx^2 + ay^2 = 0$.

11. In any system of bodies the distance of the centre of gyration from the axis of motion is a mean proportional between the distances of the centres of oscillation and of gravity from the same axis.

12. Given two sides and the included angle in a plane triangle; find the spherical angle contained by the arcs of which these sides are the chords.

13. Sum the following series:

(1). $\frac{1}{1.2.3.4} + \frac{3}{2.3.4.5} + \frac{6}{3.4.5.6} + \cdots$ to (n) terms.

(2). $\frac{1}{1.2} - \frac{3}{4.5} + \frac{5}{7.8} - \cdots$ *ad infinitum.*

(3). $\text{Sin.}\theta + \frac{1}{2}\sin.2\theta + \frac{1}{3}\sin.3\theta + \cdots$ *ad infinitum.*

(4). $\text{Sec.}\theta . \sec.2\theta + \sec.2\theta \sec.3\theta + \sec.3\theta . \sec.4\theta + \cdots$ to (n) terms.

14. A hollow cylinder of uniform thickness, and whose external diameter is given, rolls down an inclined plane of known length and inclination in a given time: required the internal diameter.

15. Let (c) be the tension at the lowest point of the catenary, then, if the co-ordinates be measured from that point, prove that

$$y = \frac{c}{2}\left(e^{\frac{x}{c}} + e^{-\frac{x}{c}}\right) - c.$$

16. Find the locus of all the points from which if pairs of tangents be drawn to a given parabola they shall intersect at right angles.

17. (1). Exhibit the complete integral of $\frac{x^{2m}\,dx}{\sqrt{(1-x^2)}}$ between the values $x = 0$, and $x = 1$.

(2). Integrate $e^{2\theta}d\theta \sin.^3\theta$: and $\frac{dx}{x^3 + 3x^2 - 4}$,

$$\text{also } \frac{dx}{dy} = \frac{a + bx + cy}{\alpha + \beta x + \gamma y}.$$

(3). Required the particular solution of the equation

$$x - y\frac{dx}{dy} = y + (a - y)\frac{dy}{dx}.$$

18. Two weights being connected by a string passing over a fixed pulley, one of them is supposed to descend vertically, and to draw up the other in the same plane along the concave circumference of a given circle: find the velocity of each at any point of the descent.

19. (1). Find the law of thickness of a string that it may hang in the form of a semicircle: and

(2). Prove that the tension, at the extremities of the horizontal diameter, is infinite.

20. A point T is given in the indefinite straight line TX on which as an axis any number of parabolas are described, so that the parameter of each may be equal to the distance of its vertex from T: required the nature of the line which shall touch all these parabolas.

21. If D, D^1 be the lengths of two degrees in two given latitudes λ, λ^1; (a) the Earth's equatorial diameter, (d) its excess above the polar diameter, prove that

$$\frac{d}{a} = \frac{1}{3} \frac{D^1 - D}{D \sin.(\lambda^1 + \lambda) . \sin.(\lambda^1 - \lambda)}.$$

22. A ball of given weight is placed at one extremity of a horizontal plane of indefinite length which has an uniform angular motion downwards round that extremity, find when the ball's velocity will be such as to make it quit the plane.

23. If (r) be one of the impossible roots of the equation

$$x^p - 1 = 0,$$

(p) being a prime number, prove that all the roots will be represented by the terms of the geometrical progression $1, r, r^2 \ldots r^{p-1}$.

24. The force of gravity being uniform, find the curve described by a projectile in a medium whose resistance varies as the velocity, and shew that when the resistance vanishes, the curve becomes the common parabola.

TRINITY COLLEGE, 1822.

1. THE specific gravity of lead is 11·324, of cork 0·24, of fir 0·45, (calling that of water 1.) How much cork must be added to 60 lbs. of lead, that the united mass may weigh as much as an equal bulk of fir?

2. A person put out to interest £2000 at 4 per cent.; he spends annually £75, and adds the remainder of his dividend to his stock; what is he worth at the end of 5 years?

3. Trace the curve whose equation is

$$y = \pm (x - a)^2 \times \sqrt{(x - b)}.$$

4. If each of two solid angles is contained by three plane angles equal to one another, each to each, the planes in which the equal angles are, have the same inclination to each other.

5. Solve the following equations:

(1). $\left.\begin{array}{l} 5x^2 + 7xy - 87 = 0 \\ 5y^2 + 7xy - 62 = 0 \end{array}\right\}$

(2). $\left.\begin{array}{l} x^2 + x - (y^2 + y) - 6 = 0 \\ x^3 + y^3 - xy(x + y) - 5 = 0 \end{array}\right\}$

(3). $x^3 - x^2 - 2x - 1 = 0$ by approximation.

(4). $8x^3 - 45x^2 + 73x - 30 = 0$ which has a divisor the form $mx - n$.

6. Sum the following series:

(1). $3 + 12 + 30 + 60 + 105 + 168 +$ &c. to (n) terms.

(2). $\frac{2^2}{1} - \frac{3^2}{2} + \frac{4^2}{3} - \frac{5^2}{4} +$ &c. *ad infinitum.*

(3). $\frac{1}{1.4.3} + \frac{1}{4.7.3^2} + \frac{1}{7.10.3^3} +$ &c. *ad infinitum.*

7. PSP' is any line (not perpendicular to the axis) passing through the focus S and terminated at P and P' in the ellipse; Prove PSP' greater than the latus rectum.

8. In stereographic projection of the sphere, the centre of the projection of any great circle is distant from the centre of the sphere by the tangent of its inclination to the plane of projection.

9. If a sphere be immersed in a fluid and its specific gravity be indefinitely less than that of the fluid, the velocity of ascent is uniform; find its value in terms of the sphere's diameter, and the accelerating force of gravity.

10. The centre of oscillation of a cone suspended by the vertex is in the base; compare the altitude of the cone with the radius of its base.

11. Investigate the expression for the surface of a solid; and find the surface of the solid generated by the revolution of the common cycloid about its base.

12. Required the chance of throwing one doublet and no more in one throw with six dice.

13. Given a very small error in declination; find the error in right ascension.

14. Prove the parallelogram of forces.

15. The force varies as the distance; required the various lines in plano along which the oscillations will be isochronous.

16. A curve is convex to its axis if the ordinate and its second differential coefficient are affected with the same sign.

17. Given $y^3 - 3y + x = 0$. Required y in terms of x; and show *à priori* the nature of the series to be assumed.

18. Transform $ax + bx^2 + cx^3 +$ &c. into a series

$$\frac{a}{1-x} + \frac{b'}{(1-x)^2} + \frac{c'}{(1-x)^3} + \text{\&c.}$$

and find the values of b', c', &c. in terms of a, b, c, &c.

19. Prove the $(m+n)^{\text{th}}$ differential coefficient of $f(xy)$ to be the same whether we differentiate m times considering x constant, and then n times considering y constant, or n times considering y constant, and then m times considering x constant.

20. Given $\frac{p_m}{q_m}$ and $\frac{p_{m+1}}{q_{m+1}}$ successive values of a continued fraction, prove $p_m q_{m+1} - p_{m+1} q_m = \pm 1$ according as m is even or odd, the first approximate being $\frac{p_0}{q_0}$: and find all the positive integral values of x and y in $13x - 17y - 54 = 0$.

21. Required the equation to the curve in which the perpendicular from the vertex on the tangent varies as the square of the ordinate; and determine the equation to the curve traced out by the extremity of the perpendicular.

22. Required the following integrals:

$$(1).\ \int \frac{dx}{x(a+bx^2)^{\frac{5}{2}}}, \quad \text{and} \quad (2).\ \int \frac{d\theta}{(a+\sin.\theta)^3 (a+\cos.\theta)^2}.$$

Integrate also the following differential equations:

(3). $adx^2 - xdy^2 = xyd^2y$.

(4). $xdy = \{y + \sqrt{(x^2 + y^2)}\} dx$.

23. A body descends in a curve towards a centre of force, and makes equal approaches to the centre in equal times. Required the equation to the curve, and the variation of the force.

24. A body (the lower part of which is a portion of a horizontal cylinder) rocks upon a horizontal plane; required the time of an oscillation.

TRINITY COLLEGE, May 1826.

1. Parallelograms upon the same base and between the same parallels are equal to one another.

2. A circle, a straight line, and a point being given in position, required a point in the line, such that a line drawn from it to the given point may be equal to a line drawn from it touching the circle. What must be the relation among the data, that the Problem may become *porismatic*, i. e. admit of innumerable solutions?

3. If A be a person's annual income, which terminates with his life, and if a be the price of assuring £100, and r be the interest of £1 for one year, what sum must he lay out annually in assurance, so that his executors may after his death receive a sum whose annual interest is equal to the reduced income?

4. In the collision of imperfectly elastic bodies, the relative velocity before impact : relative velocity after :: 1 : e.

5. Expand a^x algebraically, and prove that $a^x = e^{x \log a}$.

6. Given two sides and an included angle; required the third side in a form adapted to logarithmic computation.

7. Construct a biquadratic, and a cubic equation.

8. Find the length of the circumference of an ellipse of small eccentricity.

9. Find the diameter of curvature at the vertex of the catenary.

10. If no forces act except the resistance, and that vary as the square of the velocity, then if the times increase in geometrical progression, the velocities decrease in the same progression, and the spaces increase in arithmetical progression.

11. Suppose n halfpence to be thrown up, and those that come up heads to be taken away, and the remaining ones thrown up, and so on in the same manner till all the halfpence have come up heads; in what number of throws is it an even chance that this will take place?

12. A circular chain of given length is hung over two given points in the same horizontal line; required its position of equilibrium.

13. Given the length and elasticity of a rod standing with one end on a horizontal plane; required the greatest weight which it can support at the other end without bending.

14. Two points, connected by a rod, and acted on by gravity, slide along two given inclined planes; to determine the motion.

15. In a system consisting of any number of points moveable about an axis, a force acts to turn the system; to find the effective accelerating force on any point.

16. Explain the physical cause of solar precession, and solar nutation.

17. Sum the series,

(1). $1.2.4 + 2.3.5 + 3.4.6 + \&c.$

(2). $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots$ *ad infinitum.*

18. Find the integrals of

(1). $\frac{\sin.\theta . \cos.^2\theta\, d\theta}{1 - e^2 \sin.^2\theta}$.

(2). $\frac{d^2u}{d\theta^2} + n^2u + b\cos.(n\theta + D) = 0.$

19. Investigate the criterion of $Pdx + Qdy + Rdz$ being an exact differential; and integrate the equation

$$dx(y + z) + dy(x + z) + dz(x + y) = 0.$$

20. Draw a tangent plane to an ellipsoid, and find the equation to its normal.

21. Prove, without supposing that an equation may be resolved into factors, (1) that every equation of an odd degree must have one real root; (2) that every equation of an even one, whose last term is

negative, must have two real roots. (3) What argument may be brought to shew that the same is true, when the last term is positive? and how does it follow from all this, that an equation has as many roots as it has dimensions?

22. Let there be any number of plane areas given in magnitude and position; it is required to determine that plane on which if these areas be projected, the sum of the projections may be a maximum. Prove also that the sum of the projections on a plane perpendicular to this last will $= 0$.

23. If $A, B, C, D\ldots$ be the angular points of an equilateral polygon of m sides inscribed in a circle, whose radius $= a$, and P be any point in the circumference, then will

$PA^{2n} + PB^{2n} + PC^{2n} + \&c. = m$ times the middle term of $(1 + a^2)^{2n}$, provided n be less than m.

24. Prove the principle of least action, and apply it to determine the curve described by a body acted on by a constant force in parallel lines.

TRINITY COLLEGE, MAY 1831.

1. THE exterior angles of any multilateral figure are together equal to four right angles.

2. Prove that in extracting the square root of a number according to the common rule, when any number of figures p have been obtained, $p - 1$ at least may be added by division only. Extract the square root of 2 to six places of decimals.

3. Required the approximate ratio of $\left(2ax - x^2 + \frac{b^2x^2}{a^2}\right)^{\frac{1}{2}}$ to $(a^2 + b^2)^{\frac{1}{2}}$ when x is equal to $a + h$, and h is small compared to a.

4. Expand a^x in a series proceeding according to the powers of x.

5. Obtain a series for $(\sin. x)^n$ in terms of the sines or cosines of multiple arcs, for the several cases in which n is of the forms

$$2p,\ 4p,\ 2p + 1,\ 4p + 1,$$

6. Give a method (not tentative) of finding all the roots of an equation which are whole numbers; and apply it to the equation

$$x^4 + 2x^3 - 14x^2 + 2x - 15 = 0.$$

7. Required a solution of the biquadratic $x^4 + qx^2 + rx + s = 0$. Shew that the success of any mode of solving this equation, depends upon the circumstance, that the sums of its roots, taken two and two, are of the forms $\pm a$, $\pm b$, $\pm c$.

8. Find the general equation to the tangent at any point of a curve, when the co-ordinates are oblique. Take as an example, the ellipse, the axis of co-ordinates being parallel to conjugate diameters.

9. In a spherical triangle, having given two sides and the included angle, find the other angles.

10. Having given the three edges of a parallelopipedon and the angles they make with each other, find the solid content.

11. Determine the angle which two given planes make with each other.

12. State the principle of Monge's method of finding the equations of surfaces described after given conditions; and find the general equation of conical surfaces.

13. Find the conditions of equilibrium of any number of forces acting in any directions on a rigid body.

14. At any point of a curve surface, determine the radius of curvature of the section made by a plane passing through the normal, and express it in terms of the radii of greatest and least curvature.

15. The expansion by Taylor's Theorem of $f(x + h)$ cannot contain fractional or negative powers of h, so long as the value of x is indeterminate.

16. Obtain the differential of the area of a curve referred to polar co-ordinates.

Apply it to find the area of the loop of the curve, the equation to which is $r = \dfrac{a}{\cos.^2 \dfrac{\theta}{3}}$

17. Integrate $\dfrac{x dx}{(ax + bx^2)}$, $\dfrac{\sin.\theta d\theta}{\sqrt{(1 - \tan.^2 \theta)}}$, and $e^{a\theta} \cos.^2 \theta d\theta$.

18. State the mode in which differential equations are derived, and from the principle of derivation shew that the complete integral contains a number of arbitrary constants equal to the order of the equation.

Exemplify by eliminating m and n from $y^2 - mx - nx^2 = 0$.

19. Required the integrals of $xdy - ydx = \sqrt{(x^2 + y^2)}dy$,

$$\frac{d^2y}{dx^2} + n^2y = A\sin.nx, \text{ and } x\frac{dz}{dx} + y\frac{dz}{d} = nz.$$

20. If $f(x, y, c) = 0$ be the equation of any curve, and an indefinitely small variation be given to the parameter c, then will $\frac{d.f(x, y, c)}{dc} = 0$.

Shew that by eliminating c between these two equations, the equation of a curve is obtained, which touches all those that result by giving to c all possible values.

21. The moment of inertia of any system, with respect to any given axis, is equal to the moment about an axis parallel to this passing through the centre of gravity, together with the moment of the whole body (collected in its centre of gravity) about the given axis.

22. The centres of suspension and oscillation are reciprocal.

23. Demonstrate the principle of the conservation of *vis viva*.

TRINITY COLLEGE, MAY 1831.

1. PROVE the rule for pointing in the extraction of the square root of a numerical quantity, partly integral and partly decimal.

2. Find the number of solid feet and parts of a foot, in a rectangular solid block of wood, the edges of which are 10 feet 4 inches, 3 feet 7 inches, and 2 feet $5\frac{1}{2}$ inches in length.

3. Having given $A\sin.(x + B)$ equivalent to

$$a\sin.(x + b) + a'\cos.(x + b'),$$

determine A and B in terms of a, a', b, and b'.

4. Obtain an expression for $\tan.n\theta$ in terms of $\tan.\theta$, and shew that it remains of the same form whether n be integral, negative, or fractional.

5. If a side of a plane triangle be determined from the other sides and the included angle, and a small error (α) be committed in taking the angle, the consequent error in the determination of the side $= p\alpha$, p being the perpendicular from the angle on the side.

6. Transform $x^3 + qx + r = 0$, into an equation of the form $2z^3 - 3az^2 - a = 0$, and determine in what case the transformation is possible.

7. Shew that if $\tan.^2\theta = \frac{q^3}{27} \times \frac{4}{r^2}$, the possible root of the transformed equation in the preceding question is, $\dfrac{1}{\left(\tan.\frac{\theta}{2}\right)^{\frac{1}{3}} - \left(\cot.\frac{\theta}{2}\right)^{\frac{1}{3}}}$.

8. There are an indefinite number of spherical caps, having equal surfaces but different radii, and having a common tangent plane at their poles; it is required to determine the surface in which the circumferences of their bases are situated.

9. If the plane section of a right cone, whose vertical angle is θ, be an ellipse, the greatest and least distances of the circumference of which from the vertex are D and D'; then will the semi-axis minor of the ellipse $= \sqrt{(DD')} \sin.\frac{\theta}{2}$.

10. By means of the preceding proposition prove, that the locus of the vertices of all right cones having the same section is an hyperbola.

11. A given cylinder, the centre of gravity of which is distant by D from the axis, is placed with its axis parallel to the horizontal plane, on a given inclined plane, the friction being such as to prevent sliding: required the position of stable equilibrium; also the least value D may have, that there may be a position of rest.

12. A heavy flexible envelope exactly surrounds a sphere, and is supported by it; shew that the tension at the highest point is less than the weight of the envelope.

13. Having given $r^2 = a^2 \cos.2\theta$, the polar equation of the lemniscata, find the centre of gravity of the surface generated by the revolution of one of its loops about its axis.

14. Determine the distance of the point of contrary flexure of the curve, the equation to which is $r = a\theta^4$, from the pole of co-ordinates.

15. Obtain from the equation

$$\frac{d^2\phi}{dx^2} + \frac{d^2\phi}{dy^2} + \frac{d^2\phi}{dz^2} = 0,$$

in which the differential coefficients are partial, the value of ϕ on the supposition that it is a function of $(x^2 + y^2 + z^2.)$

16. Required the value of $\Sigma(ax^3 + bx^2)$.

17. Obtain the equation of the plane in which two given straight lines which pass through the origin of co-ordinates are situated.

18. If POP', QOQ', be two straight lines passing through any point O, and terminated by the surface of an ellipsoid, and if c, c', be the lengths of the diameters parallel to them, $\dfrac{PO \times OP'}{QO \times OQ'} = \dfrac{c^2}{c'^2}$.

19. The ultimate section of any contiguous surface by a plane, when the portion of the surface cut off is indefinitely diminished, is in general an ellipse.

20. A given cylinder unrolls from a vertical string fastened to a fixed point, and as it descends, another string, to which a given weight is attached, unwinds from it: required the accelerative forces of the two weights.

21. Determine the motion of a body on an inclined plane, the part in contact with the plane being cylindrical, and the friction being such as to prevent sliding.

Shew from the result, that the time of the small oscillations of a pendulum is very little affected by a slight defect of horizontality in the plane on which it oscillates.

22. A body projected in any direction, revolves uniformly about an axis in it, which continues horizontal and describes a parabola: determine the position of the axis about which the body, at any point of its path, appears for an instant to revolve, and shew that this axis also describes a parabola.

23. Describe geometrically the cycloid which is the brachystochronous path from a given point to a straight line making an angle of 45° with a horizontal line through the point, the motion being supposed to be from rest: and shew that if c be the distance of the intersection of this line with the horizontal line from the given point, the radius of the generating circle of the cycloid $= \frac{2c}{\pi}$.

TRINITY COLLEGE, June 1832.

1. The angle at the centre of a circle is double of the angle at the circumference upon the same base. 20. B III.

2. Reduce the fraction $\frac{3x^2 - 16x - 12}{x^3 - 8x^2 - 12x + 144}$ to its lowest terms.

3. Find the number of combinations of n things taken r and r together; and if ten letters a, b, c &c. be taken five and five together, in how many of them will a and b occur?

4. What sum of money must be placed out at simple interest, for five years, at $4\frac{1}{2}$ per cent., that the amount may be 1,000*l.*

5. The Napierian logarithm of $1 + N = N - \frac{1}{2}N^2 + \frac{1}{3}N^3 -$ &c.: prove this, and find the logarithm of 1·5 to 3 places of decimals.

6. The consecutive terms in a series of fractions converging to $\frac{a}{b}$, are alternately greater and less than the true value.

7. Find the general expression for the tangent of a compound arc, in terms of the tangents of the component arcs.

8. In a spherical triangle $\cos.A = \frac{\cos.a - \cos.b \cos.c}{\sin.b \,.\, \sin.c}$.

9. In Geodetical observations, shew how to reduce the observed to the horizontal angle.

10. Solve the equation $x^3 - qx + r = 0$ when all its roots are possible, and shew that the solution is inapplicable if the roots be not all possible.

11. Explain Newton's method of solving an equation by approximation, and find the conditions necessary to ensure accuracy in the result.

12. The areas of all parallelograms touching an ellipse at the extremities of any conjugate diameters are equal.

13. Given the general equation to a line of the second order, find the angles which its principal diameters make with the axis of x.

14. Shew how to determine the value of a vanishing fraction, and find the value of $\frac{e^x - e^{-x}}{\log.(1+x)}$ when $x = 0$.

15. Obtain the differential of the arc of a curve referred to polar co-ordinates, and find the whole length of the curve whose equation is $r = 2a \sin.^2 \frac{\theta}{2}$.

16. State the conditions that determine the order of osculation of any curve with another: also shew that the circle of curvature cannot have contact of the third order with any given curve, except its radius have arrived at a maximum or minimum value.

17. If u be a homogeneous function of x, y, z...of n dimensions, then will $nu = \frac{du}{dx}.x + \frac{du}{dy}.y + \cdots$ prove this and apply it to discover a factor which shall render $Mdx + Ndy$ a perfect differential, M and N being homogeneous functions of x and y.

18. Determine the general expressions for the co-ordinates of the centre of gravity of any curvilinear area, and find the centre of gravity of a parabolic area bounded by a line inclined to the axis at an angle α.

19. The time of descent down all chords passing through the highest or lowest points of a circle are equal.

20. In curves of finite curvature, the subtense of the angle of contact is ultimately as the square of the conterminous arc.

21. State the principle of virtual velocities, and from it deduce the equations of equilibrium of a rigid system acted upon by any forces.

22. When a heavy system is moveable about a horizontal axis, find the accelerating force upon it in any position; and also find the length of the simple isochronous pendulum.

23. The bodies in a system are acted upon only by their mutual attractions. Prove the principle of the conservation of areas.

TRINITY COLLEGE, June 1832.

1. If (a) be the first and l the last term of an arithmetic series in which the common difference is b and the number of terms n, then will the sum of the series $= \frac{n(l^2 - a^2)}{2(n-1) \cdot b}$.

2. Given that $p^2 + np$ is a perfect square, find the general form of the integral values of n.

3. $\tan.(a + b) = n \operatorname{cosec}.2a - \cot.2a$, find the value of $\tan.b$ in terms of $\tan.a$.

4. If from the extremities of the base of a semicircle, two chords be drawn crossing each other, and making angles, α, β, with the base, and if the extremities of the chords be joined, the area of the smaller of the two triangles thus formed, will equal area of larger multiplied by $\cos.^2(\alpha + \beta)$.

5. In a right-angled spherical triangle whose right angle is C, if A, c, be given, and a be required, find the error introduced by a given small error of c, and shew that when $c = 90°$ this error becomes $-\frac{1}{2} \tan.A \cdot (dc)^2$

6. If a be the possible root of the cube $x^3 - qx + r = 0$, which can be solved by Cardan's method, then must a be greater than $2\sqrt{\frac{q}{3}}$.

7. If P be any point in an ellipse whose vertex is A and nearer focus S, then will $\tan.PAS = (1 + e) \cot. \frac{ASP}{2}$.

8. Two indefinite straight lines pass through two given points, and intersect each other at right angles, if these lines be now sup-

posed to revolve, with an equal and uniform angular velocity, about the two given points, find the locus of their successive points of intersection; first, when the motions are in the same, secondly, when they are in different directions.

9. Find the distance of a given point from a given line in space.

10. The volume of the maximum spheroid inscribed in a cone equals one half the volume of the cone.

11. A sector of a circle revolves flatways round a diameter at right angles to its axis, and the volume of the solid generated equals one half the volume of the equiradial sphere, find the angle of the sector.

12. Find the equation to the conical surface, when the directrix is an ellipse in the plane of xy, and the vertex a point in the axis of z.

13. If a catenary and a parabola have a common vertex and a common axis, and if the latus rectum of the parabola, equal twice the length of the chain, whose weight expresses the tension at the lowest point, then will the arc of the catenary equal the corresponding chord of the parabola.

14. Integrate the following differentials and differential equations:

$$\frac{d\theta}{\cos.\theta}, \qquad x^2(a^2 - x^2)^{\frac{1}{2}}dx,$$

$$\frac{d^2y}{dx^2} + A\frac{dy}{dx} + By = 0, \qquad y\frac{dz}{dx} + x\frac{dz}{dy} = z.$$

15. From what point in a given line, and in what direction, and with what velocity, must a heavy body be projected, that the path of the projectile may have contact of the second order with a given circle at its highest point, the given line being a tangent to the given circle?

16. A paraboloid, the density of whose circular sections varies as their areas, stands upon its vertex on a horizontal plane, what is the length of its axis when the equilibrium is that of indifference.

17. Find the centre of percussion of a circular sector revolving flatways round its centre.

18. If a body move on a curve and be acted upon by a central force P, if R be the re-action and r, p, ρ, be respectively the radius vector, perpendicular on the tangent, and radius of curvature at any point, then will $(\text{vel.})^2 = \frac{\rho}{r}(Rr - Pp)$.

19. A sphere revolves round an axis touching its surface, find the length of the simple isochronous pendulum.

20. Find the moment of inertia of an ellipse, about one of its diameters inclined to the principal diameter at a given angle α.

21. Rays of light diverge from two equally luminous bodies given in position, shew that the surface of equal illumination with the middle point between them, is formed by the revolution of a double oval, whose area equals $\frac{1}{3}(2\pi + 3\sqrt{3})\, a^2$, (a) being the semi-distance between the bodies.

22. Find the centre of gravity of the cylindrical surface, contained between a plane perpendicular to the axis of the cylinder, and a second plane making an angle α with the first, both planes passing through one common point on the surface.

St. JOHN'S COLLEGE, 1816.

1. Given the base, the vertical angle, and, 1st the sum, 2dly the difference, and 3dly the ratio of the sides; construct the triangle.

2. Integrate the expression $\frac{t^m \dot{t}}{1 + t^2}$.

3. Investigate the motion of an air-bubble ascending in water, and shew that if x be its depth, and a the altitude of the water-barometer, its velocity is nearly $\propto \frac{1}{(x + a)^{\frac{1}{6}}}$.

4. In extracting the square root, find a limit which the remainder cannot exceed.

5. Find the time of equated payment of two sums P and p, due respectively at the end of N and n years.

6. Explain the different effects of cannon shot, and the battering-ram.

7. Integrate the equation $\frac{\dot{y}}{\dot{x}} = \frac{a + bx + cy}{e + fx + gy}$.

8. Explain the gnomonic projection, and determine the projection of a given small circle.

9. Sum the m^{th} powers of the roots of a quadratic.

10. In the ellipse or hyperbola, draw ApO parallel to any diameter PCG; then $OA \cdot Ap = PG \cdot GC$. A proof is required.

11. The strength of a rectangular beam is proportional to the breadth $\times$ (depth)^2.

12. Every equation, whose coefficients are real, may be resolved into real, simple, or quadratic factors.

13. A body revolves in an ellipse about the centre. How high will it rise, if reflected by a perfectly elastic plane directly from the centre? and how high, when the plane is not perfectly elastic?

14. A coin is chosen at random from a number of the same kind. Shew that there is a greater probability that in tossing it up, the same face will twice successively appear, than that first one, then the other, will come uppermost.

15. Determine in what part of the ocean there is but one high tide in the day.

16. Sum the series,

$$\left.\begin{array}{l} \frac{1}{1.3} + \frac{1}{2.5} + \frac{1}{3.7} + \text{\&c.} \\ \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \text{\&c.} \end{array}\right\} \text{to infinity.}$$

and, $1.2^2 + 2.2^3 + 3.2^4 +$ &c. to n terms.

17. The force varying as the distance, shew that no mutual disturbances would take place among the bodies of the universe.

St. JOHN'S COLLEGE, June 1817.

1. Find the value of 7084 lbs. at 17*s*. $10\frac{1}{4}d$. of per lb.

2. The complements of the cubes about the diameter of a cube are, all three, equal to one another.

3. Write down and explain the most rapid process you know of approximating to the circumference of a circle, and calculate it to four places of decimals.

4. Find the area of a triangle in terms of its sides.

5. Prove that $A = \sin.A . \sec.\frac{1}{2}A . \sec.\frac{1}{4}A . \sec.\frac{1}{8}A$. &c. ad infinitum.

6. One value of x in the equation $x^4 - 5x^3 - x + 5 = 0$, is 5. Find the remaining roots.

7. In the equation $x^3 - 6x^2 + 11x - 6 = 0$, one root is double another. Find them.

8. A body is projected upwards with a velocity of 85 feet per second. Where will it be at the end of half a minute?

9. Find when a body projected with a given velocity from a given point will strike a vertical plane given in position.

10. CD is drawn from the centre of the ellipse parallel to a tangent PA; ARL being parallel to BC its minor axis, prove that $AR.AL : AP^2 :: BC^2 : CD^2$.

11. A cone is filled with fluid. Compare the pressure upon its surface with the weight of the fluid.

12. Find the principal focus of a concavo-convex glass lens, placed in water, whose thickness is inconsiderable, the radii of its surfaces being 6 and 8 inches.

13. Find the least circle of aberration, into which all the homogeneal rays of the same pencil refracted by a lens are collected.

14. Find the solid content of a frustum of a cone.

15. Find the fluxion of $\frac{1+x}{\sqrt{(1-x^2)}}$, of $x \,.\, y^z$, and of $\mathrm{x} \,.\, \cos . z^n$, where $\mathrm{x} = \log . x$.

16. What arc of a circle is that which has the product of its chord and cosine a maximum?

17. Investigate the equation to the curve which has the rectangle between the rays from two fixed foci = a given square.

18. Find the law of force in the hyperbola tending to the focus of the opposite hyperbola.

19. Prove, strictly, that $\dot{v} = F\dot{t}$, and $v\dot{v} = \pm F\dot{x}$.

20. Shew that the motion of the nodes of P's orbit round T is, on the whole, retrograde.

21. Find the equation to the catenary, and give Cotes' construction.

22. Given the latitude of the place, and the Moon's declination, to find the height of the medium lunar tide.

23. $\int \frac{\dot{x}}{x^2\sqrt{(a^2+x^2)}}$, $\int \frac{x^2\dot{x}}{(2ax+x^2)^{\frac{3}{2}}}$, $\int \frac{x\dot{x}}{1-2\cos . a \,.\, x+x^2}$.

St. JOHN'S COLLEGE, June 1818.

1. Find the present worth of 897*l.* 15*s.* $6\frac{1}{2}$*d.* due nine months hence at $3\frac{1}{2}$ per cent. per annum.

2. If two straight lines be cut by parallel planes, they shall be cut in the same ratio.

3. Multiply $\sqrt[4]{(-a)}$ by $\sqrt[4]{(-b.)}$

4. In the expansion of $(a+bx+cx^2+dx^3+\&c.)^m$, write down the coefficient of x^6.

5. In the common parabola, if two parallels intersect other two parallels, the rectangles contained by the segments will be proportionals.

6. No equation can have more changes of signs than it has positive roots; nor more continuations than negative.

7. Every equation of an even number of dimensions whose last term is negative, has at least two possible roots, the one positive, and the other negative.

8. Required the number of possible roots in the equation

$$x^7 - 14x^4 + 90 = 0.$$

9. Express the roots of $x^3 - 7x + 7 = 0$ by continued fractions, and determine the accuracy of the approximation of any converging fraction deduced from these.

10. If a body be projected downwards with a velocity of 20 feet per second, find the time of describing 200 feet.

11. Two bodies projected from the top of a tower with different angles of elevation (α, β); and different initial velocities (a, b,) strike the horizontal plane at the same point; find the height of the tower.

12. If a ray of light refracted into a sphere, emerge from it after any given number of reflections, to determine the actual angle between the direction in which it is incident and emergent.

13. If the astronomical telescope be adjusted to the eye of a short-sighted person, what will be the effect upon the visual angle, when the eye is between the eye-glass and its principal focus, and when beyond it?

14. Define the term 'fluent,' and from your definition investigate $\int x^{n-1}\dot{x}$.

15. The latus rectum of a parabola $= a$, required the area cut off by a straight line which intersects the axis at the angle θ, and at the distance (b) from its vertex.

16. Required the length of the catenary.

17. Draw an asymptote to the curve whose equation is

$$y = \sqrt{\{x^2 + \sqrt{(x^4 - a^4.)}\}}$$

18. A given right cone is cut by a given plane parallel to one of it's sides; find the content of each part.

19. Find the centre of gravity of the frustum of a paraboloid whose altitude is (h), and the radii of the bases b and c.

20. If $F \propto D$ and a body be projected from a given point with a given velocity to or from a given centre of force, find the actual time of describing any given space. (*Newton*, Sect. 7. Book 1.)

21. Investigate the orbit described by a body, when projected with a given velocity, from a given point, in a given direction; supposing $F \propto \frac{1}{D^3}$.

22. A hill is in the form of a right cone. A road-way winds round it from the bottom to the top, so as to be inclined at each point at a given angle to the horizontal plane; given the dimensions of the hill, to find the length of the road-way to the top.

23. Define, and investigate expressions for the determining the centre of pressure of a plane surface, and apply them to find the centre of pressure of a semi-parabola whose extreme ordinate coincides with the surface of the fluid.

24. Explain the cause of the opposite tide.

25. Find the following fluents:

$$\int \frac{x^{\frac{1}{2}}\dot{x}}{x+1}; \quad \int \frac{\dot{x}}{(1+x^2)^m}; \quad \int \frac{\dot{x}\ \sin.x}{1+\sin.x}.$$

St. JOHN'S COLLEGE, June 1829.

1. Add together ·0125 of a pound, ·0625 of a shilling, and ·5 of a penny; and reduce 11*s*. $9\frac{1}{4}$*d*. to the decimal of a pound.

2. Prove the rule for finding the greatest common measure of two quantities, and reduce $\frac{a^6 - b^6}{a^5 - b^5 + a^2b^2(a-b)}$ to its lowest terms.

3. Extract the square roots of 654481, $1 + \sqrt{-24}$, and the cube roots of 389017, and $2 + 11\sqrt{-1}$.

4. Solve the following equations:

(1), $\frac{1}{x-2} - \frac{1}{x-4} = \frac{1}{x-6} - \frac{1}{x-8}$,

(2). $\frac{x}{3} = \frac{2}{x-1}$.

(3), $3x^{\frac{3}{2}} - x^{-\frac{3}{2}} + 2 = 0$.

(4). $\left.\begin{aligned} 9x + 8y &= 43 \\ 8x - 9y &= 6 \end{aligned}\right\}$

(5), $\left.\begin{aligned} y^2 + x &= 13 \\ y^4 + x^2 &= 97 \end{aligned}\right\}$

5. If $a : b :: c : d$, shew that $a + b : a - b :: c + d : c - d$.

6. Prove the rule for finding the sum of a geometric series, and apply it to $\frac{3}{2} + \frac{2}{3} + \ldots$ *ad inf.*

7. The three interior angles of every triangle are together equal to two right angles.

8. If the square described upon one of the sides of a triangle be equal to the sum of the squares described upon the other two sides of it; the angle contained by these two sides is a right angle.

9. One circumference of a circle cannot cut another in more than two points.

10. The angles in the same segment of a circle are equal to one another.

11. Inscribe a circle in a given triangle.

12. Ratios that are the same to the same ratio, are the same to one another.

13. Equal parallelograms, which have one angle of the one equal to one angle of the other, have their sides about the equal angles reciprocally proportional.

14. In a plane triangle, of which the sides are a, b, c, and opposite angles A, B, C, prove that

$$\frac{\sin. A}{\sin. B} = \frac{a}{b}, \qquad \cos. A = \frac{b^2 + c^2 - a^2}{2bc}.$$

15. Draw a straight line perpendicular to a plane from a given point without it.

St. JOHN'S COLLEGE, June 1829.

1. Extract the fourth root of $-a$; and prove that

$$\sqrt[3]{N} = a \cdot \frac{2N + a^3}{2a^3 + N} \text{ nearly},$$

if (a) be the integer next less than $\sqrt[3]{N}$.

2. Find the number of combinations that can be formed with (m) things, taken (r) together.

3. To describe a rectilinear figure, similar to one, and equal to another given rectilinear figure.

4. Given the three sides of a plane triangle, find its area in a form adapted to logarithms.

5. Prove that $\frac{\pi}{4} = 4 \tan.^{-1} \frac{1}{5} - \tan.^{-1} \frac{1}{239}$; and thence deduce a rapidly converging series, for the value of π.

6. Give Waring's solution of a biquadratic; and shew that the equation $$x^4 - px^3 + qx - 1 = 0,$$ has four real roots, if $\left(\frac{p+q}{4}\right)^{\frac{2}{3}} - \left(\frac{p-q}{4}\right)^{\frac{2}{3}} > 1$, and only two in the contrary case.

7. Find the general term of the expansion of $(a + b + c + \&c.)^n$ and extend the proof to the case in which the index is fractional or negative.

8. All chords of a circle terminating in the extremity of a vertical diameter are described by a heavy body in the same time, and the velocities acquired are proportional to the lengths of the chords.

9. State the principle of virtual velocities, and prove it in the case of two weights balancing upon a bent lever; and generally for a system of material points, connected among themselves by any physical means, and acted upon by any forces.

10. Apply Lagrange's theorem to shew that

$$2 \cos. n\theta = (2 \cos. \theta)^n - n(2 \cos. \theta)^{n-2} + \frac{n(n-3)}{1 \cdot 2} (2 \cos. \theta)^{n-4} - \&c.$$

and deduce the series for $\sin. n\theta$.

11. All parallelograms whose sides touch an ellipse at the extremities of conjugate diameters have the same area; and that which is equilateral, or rectangular, has respectively the greatest or least perimeter.

12. Find the polar equation to the parabola, reckoning from the focus; and thence deduce the equation

$$SY^2 = SP \,.\, SA.$$

13. When a body is immersed wholly or partly in a fluid, the horizontal pressures on its surface mutually counteract each other; and the resultant of the vertical pressures is equal to the weight of the fluid displaced, has its direction contrary to gravity, and point of application at the centre of gravity of that portion of fluid.

14. A sphere is projected vertically upwards in a fluid, find how high it will rise, and how long.

15. Find the length of the isochronous simple pendulum when a body oscillates about an horizontal axis; also determine the axes about which the time of oscillation is a minimum.

16. When a weight is raised by means of a crank, to determine the velocity; and to shew that the arm of the lever at the end of which it acts may be considered as constant, and $= \frac{7}{11}$ of the breadth of the crank.

17. Integrate the equation of differences of the first order

$$u_{x+1} - A_x u_x = B_x,$$

and sum the following series:

$$\frac{1}{1.5} + \frac{1}{3.7} + \frac{1}{5.9} + \text{\&c. to } (n) \text{ terms, and to infinity,}$$

$$2^3 . 3^2 + 2^4 . 4^2 + 2^5 . 5^2 + \text{\&c. to } (n) \text{ terms,}$$

$$\cot.^2 \frac{\theta}{n} + \cot.^2 \frac{\pi + \theta}{n} + \cot.^2 \frac{2\pi + \theta}{n} + \text{\&c. to } (n) \text{ terms.}$$

18. Express the radius of curvature of any normal section of a curve surface, in terms of the greatest and least radii of curvature and determine the radius of curvature of any normal section, at a given point of an oblate spheroid.

19. Find the differential equation of the Moon's motion in titude.

20. Assuming the principle of least action, apply the calculus of ıriations to find the curve described in one plane by a body attracted a fixed centre of force.

St. JOHN'S COLLEGE, Dec. 1829.

1. If from the ends of the side of a triangle there be drawn ro straight lines to a point within the triangle, these shall be less an the other two sides of the triangle, but shall contain a greater ıgle. (*Eucl.* I. 21.)

2. If two similar parallelograms have a common angle, and be nilarly situated; they are about the same diameter. (*Eucl.* VI. 26.)

3. Planes to which the same straight line is perpendicular, are ırallel to one another. (*Eucl.* XI. 14.)

4. ABC is an equilateral triangle; E any point in AC; in BC :oduced take $CD = CA$, $CF = CE$; AF, DE intersect in H;

$$\frac{HC}{EC} = \frac{AC}{AC + EC}.$$

5. Given the base, the ratio of the sides containing the vertical ngle, and the distance of the vertex from a given point in the base;) construct the triangle.

6. The circumference of the circle ACE is divided into six equal ırts in the points A, B, C, D, E, F; G is the centre of ACE; ith radius AC, and centres A, D, describe two circles intersecting H; with radius HG, and centres C, E, describe two circles inter- :ting in K; $AK.AG = KG^2$.

7. ABC is an isosceles triangle; draw CE perpendicular to the se AB; draw ADF intersecting CE in D, and CB in F;

$$\frac{DE}{CE} = \frac{CA - CF}{CA + CF}.$$

8. Find a point in a given straight line, such that the sum of distances from two given points, (not in the same plane with the ren straight line) may be the least possible.

9. BEF is a circle inscribed in the triangle ABC, touching the sides BC, AC, AB, in D, E, F; HIK is a circle touching AB in K and CB, CA produced in H, I; in CH take $CL = CA$, and in CI take $CM = CB$;

$$FK = AM, \text{ and } 4 . AF . FB = AL . MB.$$

10. Find a point, such that the perpendiculars let fall from it on three straight lines given in position, may be in a given ratio.

11. BE, AC are parallel lines; F, G, H, &c. a series of equidistant points in AC; draw Bfh cutting BE in B, and EF, EG, EH &c. in f, g, h &c.; Bf, Bg, Bh &c. are in harmonic progression.

12. With any point A in the circumference of the circle ABF as a centre, and any radius AB, less than the diameter, and greater than half the radius of ABF, describe a circle cutting ABF in F; in it place the chords BD, DE, EC, each equal to the chord AB; with radius CF, and centres A and C, describe two circles intersecting in G; with the same radius, and centre G, describe a circle intersecting the circle BDF in H; the chord HB is equal to the radius of the circle ABF.

St. JOHN'S COLLEGE, Dec. 1830.

1. Parallelograms upon the same base and between the same parallels are equal to one another.

2. Of unequal magnitudes the greater has a greater ratio to the same than the less has.

3. If the diameter of a circle be one of the equal sides of an isosceles triangle, the base will be bisected by the circumference.

4. The line joining the centres of the inscribed and circumscribed circles of a triangle, subtends at any one of the angular points, an angle equal to the semi-difference of the other two angles.

5. Find a point without a given circle, such that the sum of the two lines drawn from it touching the circle, shall be equal to the line drawn from it through the centre to meet the circle.

6. If a circle roll within another of twice its size, any point in its circumference will trace out a diameter of the first.

7. If from any point in the circumference of a circle, a chord and tangent be drawn, the perpendiculars dropped upon them from the middle point of the subtended arc, are equal to one another.

8. If α, β, γ, represent the distances of the angles of a triangle, from the centre of the inscribed circle, and a, b, c, the sides respectively opposite to them, then $\alpha^2 a + \beta^2 b + \gamma^2 c = abc$.

9. Describe a circle through a given point, and touching a given straight line, so that the chord joining the given point and point of contact, may cut off a segment capable of a given angle.

10. Shew that the perimeter of the triangle formed by joining the feet of the perpendiculars dropped from the angles upon the opposite sides of a triangle, is less than the perimeter of any other triangle, whose angular points are on the sides of the first.

11. Explain what is meant by the equation to a curve; find the equation to a straight line, and state clearly the meaning of the constants involved.

12. Trace the circle whose equation is

$$a(x^2 + y^2) + b^2(x + y) = 0;$$

draw the lines represented by the equation

$$y^2 - 2xy \sec.\alpha + x^2 = 0,$$

and shew that the angle between them is α.

13. The portion of a straight line intercepted by two rectangular axes, and the perpendicular upon it from their intersection, are each of a given length; what is the equation to the line?

14. Find the equation to the ellipse, and deduce that to the parabola from it

15. Find the co-ordinates of the point from which if three lines be drawn to the angles of a triangle, its area is trisected.

16. In the last question, the distance, from the angle A, of the required point $= \frac{2}{9}(b^2 + c^2 - a^2)$.

17. If the centre of the inscribed circle of a triangle be fixed, and α, β, γ, represent the distances of its angles from any fixed point in space, then whatever position the triangle assumes, the expression $\alpha^2 a + \beta^2 b + \gamma^2 c$ is invariable.

St. JOHN'S COLLEGE, May 1831.

1. The interest of £847 1s. 8d. for 2 years and 4 months is £23. 5s. $10\frac{3}{4}$d., what is the rate per cent. at simple interest?

2. What is the value of £·9545454...? Extract the square roots of $31 + 10\sqrt{6}$ and of $3 - \sqrt{-7}$.

3. What is the area of a room 17 feet 5 inches long, and 10 feet 11 inches wide?

4. The reciprocals of quantities in harmonic progression are in arithmetic progression. Insert two harmonic means between 3 and 4.

5. Prove the Binomial Theorem for all values of the index: and write down the first four terms of the expansion of $\dfrac{1}{\sqrt{(x^2 - 1)}}$.

6. Solve the following equations:

(1). $\dfrac{5x + 19}{9} - \dfrac{7 - 2x}{5} = \dfrac{4}{3}(x + 4)$

(2). $\dfrac{\sqrt{x} + \sqrt{(a - x)}}{\sqrt{x} - \sqrt{(a - x)}} = 2$

(3). $a + bx + cx^2 = 0$

(4). $$\left.\begin{array}{l} \frac{3}{4}\sqrt{(x + y)} = 1 + \dfrac{1}{\sqrt{(x + y)}} \\ \sqrt{(x + y)} + \sqrt{(x - y)} = 5 \end{array}\right\}$$

7. The sum of the interior angles of any rectilinear figure together with four right angles is equal to twice as many right angles as the figure has sides: and the sum of the exterior angles is equal to four right angles.

8. If a regular polygon have n sides, any one of its interior angles $= \dfrac{n - 2}{n} \cdot \pi$.

9. One circle cannot touch another in more points than one.

10. Describe an isosceles triangle having each of its angles at the base double of the third angle.

11. Deduce from the last proposition the value of $\sin.18^\circ$.

12. If a whole magnitude be to a whole as a magnitude taken from the first is to a magnitude taken from the second, the remainder is to the remainder as the whole is to the whole. Book 5, Prop. XIX.

13. If a straight line be at right angles to a plane, every plane, which passes through it, is at right angles to that plane.

14. Explain the respective advantages of Briggs' and Napier's systems of logarithms. Prove that $\log._e N = \log._e 10 \,.\, \log._{10} N$.

15. Prove that $\tan.A = \pm \sqrt{\left(\dfrac{1 - \cos.2A}{1 + \cos.2A}\right)}$

$$(\cos.A \pm \sqrt{-1}\,\sin.A)^m = \cos.mA \pm \sqrt{-1}\,\sin.mA.$$

16. Given two sides of a triangle and the angle included by them, find the third side in a form adapted for logarithmic computation.

17. Draw a straight line perpendicular to two given straight lines in space.

St. JOHN'S COLLEGE, May 1831.

1. If four magnitudes of the same kind be proportionals, the greatest and least of them together are greater than the other two together.

2. If a straight line be at right angles to a plane, every plane which passes through it shall be at right angles to that plane.

3. Sum the following series:

$$1 + 2 + 3 + 4 + 5 + \cdots + n$$

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \cdots + n^2$$

and from them deduce the sum of the series

$$1.2 + 2.3 + 3.4 + 4.5 + \text{\&c. to } (n) \text{ terms.}$$

4. Solve a biquadratic equation by Waring's method, and shew that the solution can only be applied to those cases in which two roots are possible and two impossible.

5. Solve the recurring equation

$$x^5 - \frac{15}{2}x^4 + \frac{37}{2}x^3 - \frac{37}{2}x^2 + \frac{15}{2}x - 1 = 0;$$

and if $x = y + 2y^2 + 3y^3 + \&c.$, find y in terms of x by reversion of series.

6. Find the number corresponding to a logarithm not found exactly in the tables.

7. Find the equation to a plane.

8. In the parabola, prove that $SP.SA = SY^2$; and if ordinates PN, pn, be drawn from the extremities of any parameter PSp, shew that $AN : AS :: AS : An$.

9. Find the equation to the section of an oblique cone.

10. Express the sine and cosine of an angle of a plane triangle in terms of its sides.

11. When a body is supported upon a curve, situated in a vertical plane; to find the conditions of equilibrium.

12. Find the equation to the catenary, when the chain is extensible; and from it deduce that of the catenary when inextensible.

13. State and prove the principle of the conservation of *vis viva.*

14. Determine the density of the atmosphere at any altitude above the Earth's surface, supposing the force of gravity to vary inversely as the square of the distance from the Earth's centre.

15. Prove the principle of virtual velocities in the case of a fluid mass acted upon by any forces.

16. When a body is acted upon by any forces, prove that the motion of the centre of gravity will be the same as if all those forces acted at the centre; and the motion of rotation the same as if the centre of gravity were fixed, and the same forces were applied.

17. Find the attraction of a spherical shell on a corpuscle without it; the several particles of the shell attracting with forces, which vary as $\frac{1}{(\text{dist.})^2}$.

18. Investigate an expression for the n^{th} difference of any function, in terms of its successive values; and prove that

$$\Delta . \cot. 2^x\theta = -\frac{1}{\sin. 2^{x+1}\theta}.$$

19. Find the integrals of

$$x^2 + x + 1, \quad \frac{x^2}{2^x}, \quad \frac{1}{(2x-1)(2x+5)},$$

and sum the series

$$\frac{1}{\sin.\theta} + \frac{1}{\sin.2\theta} + \frac{1}{\sin.2^2\theta} + \text{\&c.} + \frac{1}{\sin.2^{x-1}\theta},$$

and also the recurring series

$$1 + 2 + 6 + 22 + 86 + \text{\&c. to } (x) \text{ terms.}$$

20. Expand $\frac{1}{1+e\cos.\theta}$ in a series of cosines of the multiples of θ.

21. Find the curve of quickest descent from one given point to another given point.

22. A rod of given length slides by gravity between a horizontal and vertical line. Find the angular velocity of the rod in any position; and shew that the curve, which the rod always touches, is a hypocycloid, having the radius of its generating wheel $= \frac{1}{4}$ of the radius of the globe.

St. JOHN'S COLLEGE, June 1832.

1. The rectangle contained by the diagonals of a quadrilateral inscribed in a circle is equal to the rectangles contained by its opposite sides.

2. Draw a straight line perpendicular to two given straight lines not in the same plane.

3. Shew that $a^m . a^n = a^{m+n}$, for all values of m and n.

4. Insert n arithmetic means between a and b, and sum the series $1^2 + 2^2 + 3^2 + \cdots + n^2$.

5. Prove the formulæ

$$\sin.A + \cos.A = \pm\sqrt{(1 + \sin.2A)}$$
$$\sin.A - \cos.A = \pm\sqrt{(1 - \sin.2A)},$$

and explain for what values of A the different signs are to be used.

6. Investigate a formula for determining the ratio of the circumference of a circle to the diameter.

7. An odd number of the roots of $X' = 0$ lies between each adjacent two of $X = 0$.

8. Find by Trigonometry the roots of $x^3 - \frac{3x}{2} = \frac{1}{2}$.

9. Express $(a^2 - 2aa' \cos.\phi + a'^2)^{-\frac{1}{2}}$ in a series of the form

$$A_0 + A_1 \cos.\phi + A_2 \cos.2\phi + \&c.,$$

and shew that $A_{n+1} = \frac{2n}{2n-1} \frac{a^2 + a'^2}{aa'} . A_n - \frac{2n+1}{2n-1} . A_{n-1}$.

10. In the ellipse $CP^2 + CD^2 = AC^2 + CB^2$.

11. Investigate the equation to the section of a right cone made by a plane, and shew in what cases it is a parabola, ellipse, or hyperbola.

12. How are force and velocity measured; and how is the numerical value of the force of gravity determined? When a body is projected vertically upwards with a given velocity, find its velocity after ascending through a given space.

13. Find the condition of equilibrium that a body may be kept at rest by given forces on a curve surface.

14. When a body is sustained between two fluids, compare the parts immersed in each.

15. Find the density of the air at any height above the Earth's surface, gravity varying as $(\text{distance})^{-2}$ from the Earth's centre.

16. If the course of a ray of light successively reflected at two plane mirrors be in one plane, the deviation equals twice the inclination of the mirrors.

17. A series of waves of homogeneous light diverges from a given point, and is reflected by two plane mirrors, which are nearly in the same plane with each other. Shew that the appearance upon a screen placed parallel to the intersection of the two mirrors is a series of black and white bars.

18. Supposing the Earth a homogeneous spheroid, shew that the force of gravity at any point is proportional to the normal at that point. Mention the different methods by which the Earth's ellipticity is determined. From what observations does it appear that the Earth is not homogeneous?

19. Prove that $\Delta . 2^x . \tan.^{-1} \frac{\theta}{2^x} = 2^x . \tan.^{-1} \frac{\theta^3}{2^x . (2^{2x+2} + 3\theta^2)}$, and integrate the equation, $u_{x+1} u_x - a(u_{x+1} - u_x) + 1 = 0$.

20. If $R = \frac{m' . (xx' + yy')}{(x'^2 + y'^2)^{\frac{3}{2}}} - \frac{m'}{\{(x - x')^2 + (y - y')^2\}^{\frac{1}{2}}}$, shew that the equations of motion of a disturbed planet are

$$d_t^2 x = -\frac{\mu x}{(x^2 + y^2)^{\frac{3}{2}}} - d_x R, \quad d_t^2 y = -\frac{\mu y}{(x^2 + y^2)^{\frac{3}{2}}} - d_y R;$$

in the solution of these equations explain what is meant by the expression $\int_t d_t (R)$.

21. What are meant by the periodic and secular variations in the planetary orbits, and on what quantities do they respectively depend? Are all the elements of a planet's orbit subject to secular variation? Explain generally the method which Lagrange adopted for obtaining the irregularities of the disturbed orbit from the elements of the undisturbed ellipse.

22. Prove the principle of the conversation of *vis viva*, and shew that the whole *vis viva* = the *vis viva* from translation + the *vis viva* from rotation.

CAIUS COLLEGE, Dec. 1826.

1. At what time after eight o'clock, are the hour and minute hands of a clock first at right angles to each other?

2. What will a deal board, 3 feet 7 inches broad, and 10 feet 3 inches long cost, at 1*s*. 3*d*. the square foot?

3. Prove that $m . (m^2 - 1) . (m^2 - 4)$ is divisible by 15.

4. If the angles of the figure in *Euc.* I. 47 be joined, the sum of the squares of the lines bounding the figure, will equal eight times the square of the hypothenuse.

5. If three circles mutually intersect, and the corresponding points of intersection be joined, the three straight lines will meet in one point.

6. If perpendiculars be drawn to two lines given in position from a point P, and the distance between the feet of the perpendi-

culars be a constant quantity; the locus of P is a circle, whose centre is the intersection of the two straight lines.

7. Sum of the tangents BA, AC is a constant quantity; prove that the locus of A is an ellipse; also, find the relation of the constants, that the foci may be in the centres of the circles.

8. If an ellipse and hyperbola be described on the same axes, and any line be drawn from the centre cutting them, the tangents at the points of intersection will make equal angles with the axes.

9. If a, b, c be the sides of a triangle, the sum of the squares of the distances of the centre of the inscribed circle from the angular points $= ac + bc + ab - \dfrac{6abc}{a+b+c}$.

10. If $\sin.4\theta + 4\sin.3\theta.\cos.\theta = 0$; find $\tan.\theta$.

11. Solve the equations

$$x^4 + 6x^3 - 14x^2 - 14x - 3 = 0,$$
$$x^2 - 4.(1+\sqrt{3}).x + 3 = 0,$$

and give the roots of the latter in the form of binomial surds.

12. If $u_x = x^n$, prove that

$$u_x - x.\Delta u_x + x.\frac{x+1}{2}.\Delta^2 u_x - \cdots = 0$$

13. If a, b, c be any quantities not all equal, prove that

$$abc > (a+b-c).(a+c-b).(b+c-a)$$

14. There are two infinite geometrical series, the first terms of each being unity; if s_1 and s_2 represent their sums, prove that the sum of the infinite series formed by multiplying together the corresponding terms $= \dfrac{s_1.s_2}{s_1+s_2-1}$.

15. Prove that $\sin.a - 3\sin.2a + 5\sin.3a - \cdots$ to x terms

$$= \frac{(-1)^{x+1}}{4\cos.^2\frac{a}{2}}.\{(2x+1).\sin.xa + (2x-1).\sin.(x+1).a\} - \frac{1}{2}\tan.\frac{a}{2}.$$

CAIUS COLLEGE, Dec. 1827.

1. Prove that the sum of any quantity and its reciprocal is not less than 2.

2. Demonstrate that $ab + cd < \sqrt{(a^2 + d^2)} . \sqrt{(b^2 + c^2)}$.

3. In how many years will the interest due upon £.100 be equal to the principal, allowing compound interest?

4. Prove that $\sin.(a + b) . \sin.(a - b) = \sin.^2 a - \sin.^2 b$.

5. Given the logarithms of 3 and 7: find the logarithm of 3087.

6. The sum of four numbers in arithmetical progression is 14, and the sum of their squares is 54: what are the numbers?

7. Solve the cubic equation $x^3 + 4x^2 - 10x - 4 = 0$.

8. Sum to infinity the series whose general term is

$$\frac{x^2 - 2}{x.(x+1)} . \frac{1}{2^x}.$$

9. The circumference of a circle is a mile: find the side of a square whose area is equal to that of the circle.

10. If any straight line ASP be drawn through the focus S of an ellipse; prove that $\frac{1}{SA} + \frac{1}{SP}$ is a constant quantity.

11. If CE, AD bisect the sides of the triangle ABC, the area AOC = area $BEOD$.

12. A person sells a quantity of sugar at two guineas per cwt., making a profit of 5 per cent., and clears by the whole £.100: at what price ought he to have sold it per cwt., to have cleared £.200?

13. The area of a regular polygon inscribed in a circle, is to the area of a similar one circumscribed about the circle as 2 to 3: find the number of sides on the polygon.

14. If a section of a conoid be made by a plane passing through one of its foci, that focus is also one of the foci of the section.

CAIUS COLLEGE, Dec. 1827.

1. The interior of two circles which touch internally is taken away, and the remainder vibrates in its plane round an axis passing through the point of contact. Required the time of a small vibration.

2. Given the difference of the lengths of the longest and shortest days. Required the latitude of the place.

3. A solid is generated by the revolution of the catenary about a tangent at the vertex. Required the content.

4. If two sides of a spherical triangle = half a circle, the arc drawn from the vertex bisecting the base equals a quadrant.

5. The square of any plane = sum of the squares of its projections on the co-ordinate planes.

6. The maximum paracentric velocity in an ellipse, center of force in focus, $= \frac{ce}{a(1-e^2)}$, $\frac{c}{2}$ being the area described in 1″.

7. Q hanging freely supports P on the parabola APC, whose axis AB is horizontal, by means of a string passing through the focus; find the position of equilibrium.

8. An ellipse is placed with its axis major vertical; find the radius vector drawn from the upper focus down which the time is a minimum and determine the limits of the problem.

9. A body is projected perpendicularly upwards within the spherical surface ADB from the extremity A of the horizontal diameter. Required the initial velocity that the body may, after leaving the surface, pass through C.

10. A body falls from the top of a tower whose height is a. And another is projected upwards with a velocity a at the same instant from the base. Where will they meet?

11. The centripetal force is greater or less than the centrifugal according as the radius vector is greater or less than half the latus rectum; the centre of force in focus.

12. Required when that part of the equation of time arising from the obliquity of the ecliptic is a maximum, supposing the sun to move uniformly.

13. At the first observation $NPD = 76°$, $Z = 48°.26'.48''$. At second observation $Z = 39°.58'.48''$; half elapsed time in degrees $= 11°.15'$, and change of $NPD = 82''$. Required the latitude.

14. A solid of revolution is formed by the catenary about its axis, and a hemisphere is placed on it: find the radius of the hemisphere where stable equilibrium ends and unstable begins.

15. A body whose elasticity is e is projected from a perfectly hard horizontal plane with a given velocity and in a given direction, shew that the successive ranges are in geometric progression—and find the point where the body begins to roll.

16. Two beams of given length rest with one extremity against two perfectly smooth vertical planes; and with their other extremities against each other on the same perfectly smooth horizontal plane: find the position of equilibrium.

CAIUS COLLEGE, Dec. 1829.

1. A Person invests £.10,000 in the 3 per cents, when they are at 75; they afterwards rise to 78, when he sells out and invests the produce in the 4 per cents at 105; what is the annual income derived from the latter stock?

2. 17 cwt. 3 qrs. 22 lbs. at £4. 6*s*. $7\frac{1}{2}$*d*. per cwt.

3. The sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of the sides.

4. The sum of two quantities is s_1, the sum of their cubes is s_3; find the sum of their squares.

5. Solve the equations.

$$(1).\quad \left.\begin{aligned} y^2 + y + 17x &= 54. \\ \sqrt{(x^2 + 2y^2)} + x &= 8 \end{aligned}\right\}$$

$$(2).\quad \left.\begin{aligned} x^2 y^3 z^4 &= a \\ x^3 y^4 z^2 &= b \\ x^5 y^2 z^3 &= c \end{aligned}\right\}$$

6. Find $\sqrt{(35)}$ in a continued fraction.

7. Sum $1^2 . 2^2 + 4^2 . 5^2 + 7^2 . 8^2 +$ &c. to n terms.

8. Find the condition necessary that a plane may pass through a given straight line and be perpendicular to another straight line not in the same plane with the former.

9. If three arcs are in arithmetic progression, the sum of the sines of the extremes is less than twice the sine of the mean arc.

10. A, B are two fixed points; if two straight lines AC, BC be drawn making a given angle C; prove that the straight line bisecting C passes through a fixed point and determine the point geometrically.

11. The sum of nine terms of the series

$$n^2 + (n + 1)^2 + (n + 2)^2 + \ldots\ldots$$

is equal to 501; find the value of n.

12. Find u_x from the equation

$$u_x + 2 \cos.\theta \, . \, u_{x-1} + u_{x-2} = 0,$$

$$u_1 \text{ being} = \cos.\theta \text{ and } u_2 = - \cos.2\theta$$

13. If perpendiculars be raised upon the middle points of the sides of a triangle and respectively equal to half those sides and the extremities of the perpendiculars joined; the sum of the squares of these last lines is equal to the sum of the squares of the sides of the triangle + six times its area.

14. If any tangent be drawn to an ellipse, and four perpendiculars be drawn to the axis major from the centre of the ellipse, the two extremities of the axis major, and the point of contact; these four perpendiculars are proportional.

15. If S_x represent $\alpha^x + \beta^x$; prove that

$$S_x = \frac{2S_{x-1} . S_{x-2} . S_{x-3} - S^2_{x-1} . S_{x-4} - S^3_{x-2}}{S^2_{x-3} - S_{x-2} . S_{x-4}}$$

CAIUS COLLEGE, Dec. 1829.

1. A GIVEN cone is placed with the circumference of its base on a prop, determine the force which must act vertically at the vertex to keep it at rest with its axis in a given position.

2. The centre of gravity of a solid of revolution is at the distance of $\left(\frac{1}{m}\right)^{\text{th}}$ part of the axis from the vertex. Required its form.

3. Divide a pyramid into two parts by a plane parallel to the base, so that the distance of the centre of gravity of the upper part from the vertex may be equal to three times that of the lower from the base.

4. A semicycloid has its vertex downwards and base horizontal. Divide it into three parts so that the times of descent of a heavy body through them may be in the proportion of 1, 2 and 3.

5. A body, of elasticity e, is projected in vacuo from the horizontal plane in a given direction and with a given velocity, find the whole area contained by the path described and the horizontal line before the body rests; also the whole time of motion.

6. A pendulum at the distance of $\left(\frac{1}{k}\right)^{\text{th}}$ part of the Earth's radius above its surface, and another at the same distance below its surface vibrate in the same time. Compare their lengths.

7. A heavy body descends in a semicycloid, whose vertex is downwards and base horizontal, and the motion commences from the highest point; in every ordinate MP, MQ is taken proportional to the pressure at P. Required the locus of Q.

8. Required the distance of a straight line from a spherical reflector, that the image may be an ellipse whose major-axis is equal to twice its minor axis.

9. The caustic by refraction of a plane surface, for rays diverging from a point, is the evolute of an hyperbola or ellipse according as the passage is into a denser or a rarer medium.

10. In a glass lens, find the relation between the radii, which with a given focal length and aperture will make the aberration for parallel rays a minimum.

11. When the top of the mast of a ship has just come above the horizon, two complete images of the ship, vertical to it, one inverted and the other erect, have been observed in the air by a person on shore. Can you account for this phenomenon?

12. $u = (\tan.^{-1}x)^n$, find $\dfrac{d^n u}{dx^n}$.

13. Draw all the asymptotes to the curve whose equation is

$$Ax^3 + B^2x^2 + C^3x + D^4 + E^3y - Fxy^2 = 0$$

14. Transform the equation

$$\frac{d^2u}{dr^2}\left(\frac{dr^2}{dx^2} + \frac{dr^2}{dy^2}\right) + \frac{du}{dr}\left(\frac{d^2r}{dx^2} + \frac{d^2r}{dy^2}\right) + \frac{d^2u}{dz^2} = 0$$

where $u = f(r.z)$ and $r = \phi(x.y)$ into one where $u = \psi(x.y.z)$.

15. Integrate $\dfrac{d\theta}{(\cos.\theta)^4}$, $\dfrac{dx}{1 + x^4}$,

$\dfrac{dy}{dx} + \dfrac{y^2 + y + 1}{x^2 + x + 1} = 0$, an algebraical result is required.

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x,$$

$$x\frac{dz}{dx} + y\frac{dz}{dy} = \frac{xy}{z},$$

$$\frac{dz}{dx}\cdot\frac{dz}{dy} = \frac{z}{a}.$$

16. Investigate the differential expression for the volume of any solid referred to three rectangular co-ordinates, and apply it to find the volume of an oblate spheroid.

17. Let AB be the quadrant of a circle whose radius is (1) and centre S, and let any radius SQ be produced both ways to P, P' so that QP, QP' be each equal to the secant of AQ, then if α, α' be the areas described by SP, SP' respectively, and θ be the $\angle ASQ$ passed over, prove that

$$\theta = \tan.^{-1}\frac{e^{\alpha-\alpha'} - e^{-(\alpha-\alpha')}}{2}.$$

18. A curve of double curvature is generated by a point moving uniformly on a quadrant from the pole to the base, whilst the quadrant itself moves uniformly through 90°. Deduce the equations to its osculating plane.

19. Divide an elliptic quadrant into two parts, so that their difference shall be equal to the difference between the semi-axes.

20. Integrate the equation of differences,

$$u_{x+2} + A(x + 2)u_{x+1} + B(x + 1)(x + 2)u_x = 0.$$

CAIUS COLLEGE, MAY 1830.

1. A BEAM fixed at one extremity to a hinge leans with the other against the slant side of a right-angled wedge placed on a smooth horizontal table; what force must be applied at the back of the wedge to keep it at rest?

2. Find the centre of gravity of an arc of the common helix.

3. Prove that the locus of the centres of gravity of all cones which have a common base is similar to the locus of their vertices.

4. Two strings of different, but uniform densities and acted on by gravity, are fixed at their upper extremities, and joined at the lower; prove that at the junction the curvature of the two catenaries are as the densities.

5. A series of perfectly elastic balls are such that

$$(m+1)^{\text{th}} \text{ ball} : m^{\text{th}} :: (2m)^2 + 1 : (2m)^2 - 1;$$

given the velocity of the first ball, find the velocity communicated to the last when their number becomes infinite.

6. The sum of the angular velocities of a projectile round the focus, when the projectile is at opposite extremities of any chord drawn through the focus, is constant; required proof.

7. Prove that the image of the curve, of which the equation is

$$\frac{y}{x-r} = \sqrt{\frac{x}{2r}},$$

formed by a very small spherical mirror (rad. r) passing through the origin at right angles to the axis of x, coincides with the curve itself.

8. Find the relation between the conjugate foci of a double convex lens, the upper surface of which is in air, the lower immersed in water.

9. Explain fully the cause of the two coloured and one uncoloured image of the Sun formed by a prism. In what position of the prism will the fainter spectrum vanish?

10. Find the n^{th} differential coefficient of $x^n . \log.(x)$.

11. Trace the curve of which the equation is
$$a^2(x^2 - y^2)^2 = (x^2 + y^2)^3,$$
and shew that the entire area $= \frac{\pi a^2}{2}$.

12. Integrate $\frac{dx}{x}\log_e(1 - x^n)$, from $x = 0$ to $x = 1$,
$$\frac{d\theta}{\sin.\theta + \cos.\theta},$$
$$\frac{dx}{x} + \frac{dy}{y} = \epsilon^{xy}.\frac{x^m dx}{y^2},$$
$$\frac{d^2u}{d\theta^2} + u = a.$$

13. A cone having its vertex without the surface of a given sphere and its axis passing through the centre of the latter, intersects the sphere, determine the content of the solid common to both.

14. Find the orthogonal trajectory to a series of circles which have a common chord.

15. Find an expression for the curvature at any point of a curve of double curvature when the arc is the independent variable.

16. If $f(x), f'(x), f''(x)$ &c. be any function of x and its successive differential coefficients, then shall
$$f(x) = f(x+h) - f'(x+2h).\frac{2h}{1.2} + f''(x+3h).\frac{3^2h^2}{1.2.3}$$
$$-f'''(x+4h).\frac{4^3h^3}{1.2.3.4} + \text{\&c.}$$

CAIUS COLLEGE, June 1832.

1. Required the value of 7 cwt. 3qrs. 27lbs. at £3. 13s. $3\frac{1}{2}d.$ per cwt.

2. Solve the equations $\sqrt{3}.x^2 - 6x + \sqrt{3} = 0$, and
$$\cdot 03x^2 + 3x = 20.$$

3. A, B, C travel from the same place at the rate of 4, 5, and 6 miles an hour respectively: B starts two hours after A: how long after B must C start that they may both overtake A at the same time?

4. When are the minute and second hands of a watch first together after 12 o'clock?

5. Find the fraction equivalent to the circulating decimal $\cdot\dot{0}12345679\dot{0}$, and reduce it to its lowest terms.

6. Find the fourth term of the expansion of $(x\sqrt{2} + y\sqrt{3})^6$ to three places of decimals.

7. Extract the square root of $\dfrac{2}{2 - \sqrt{3}}$ in the form of a binomial surd.

8. What annuity is equivalent to a sum of £200 paid at the end of every two years, the rate of interest being 5 per cent. per annum? Shew from general reasoning that it is less than £100.

9. Find the equation to a straight line of which the distance from the origin is a, and the part between the axes of co-ordinates b.

10. Prove that $\dfrac{1.4}{2\ 3} + \dfrac{2.5}{3.4} + \dfrac{3.6}{4.5} + \ldots$ to x terms $= \dfrac{x.(x+1)}{x+2}$.

Find the sum of x terms of $a + 3a^2 + 7a^3 + 15a^4 + \ldots$

and of $1^2 + 3^2 + 7^2 + 15^2 + \ldots$

11. Prove that $\sin.3A = 4\sin.(60 - A)\sin.A \,.\, \sin.(60 + A)$.

12. The angle of elevation of a tower situated on a plane inclined to the horizon at an angle θ is α, and at a station distant a from the former the elevation is β, the stations being on the inclined plane; find the height of the tower.

13. The sum of the squares of the distances of one angular point of a regular polygon of n sides from all the others $= 2nr^2$, where r is the radius of the circumscribing circle.

14. If $(x_1 . y_1)$, $(x_2 . y_2)$, $(x_3 . y_3)$ be the co-ordinates of the angular points of a triangle, twice its area

$$= (x_3 - x_2)(\quad - y_2) - (y_3 - y_2)(x_1 - x_2).$$

15. If from any point A without an ellipse tangents AB, AC be drawn and BC joined; then if any line be drawn from A cutting the ellipse in D, E, the tangents from E and D will meet in BC produced.

CAIUS COLLEGE, JUNE 1832.

1. A CERTAIN quantity is to be weighed out by a false balance: by weighing at both ends, and taking half the weights for the required quantity, $\frac{2}{7}$ of an ounce is gained in every pound, find the error in the place of the fulcrum.

2. A weight Q is supported on a hexagonal axle and wheel, of which the radius has to the radius of the circumscribing cylinder of the axle, the ratio $\sqrt{3} : 1$. The difference between the greatest and least forces required to maintain the equilibrium

$$= \frac{Q}{2} (\sec. \frac{\pi}{6} - 1).$$

3. The effect of lengthening the arm of a balance, will be counteracted by allowing the weight suspended from that arm to rest upon a catenary whose axis is in the same straight line with the beam, and which has a tension at its vertex, equal to a weight of chain of half the length of the beam.

4. A given isosceles triangle whose altitude is $\frac{3}{2}$ rad. of a circle stands upon the interior of the circumference, find the arc through which it can be moved without being overturned, and the pressures on the circumference in the extreme position of equilibrium.

5. A cone of which the density varies as the distance from its vertex will balance on the centre of its axis, if a weight $= \frac{3}{5}$ its own weight be suspended at the vertex.

6. A bomb vessel is sailing in a given direction; a shell discharged with known velocity from a mortar of a certain elevation, is observed to strike a fort at the instant the ship is opposite to it: find the uniform rate of the vessel.

7. In making experiments upon the weights of bodies at different altitudes, the difference of height is found to vary nearly as the difference in the weight.

8. A ball of given elasticity is struck directly from the middle to the side of a billiard table; find the velocity which is communicated to it, that after the third impact, it may just reach the opposite side; considering friction as a uniform force.

9. The locus of the point to which the descents from two given points in a vertical line are isochronous, is the rectangular hyperbola.

10. An object is placed between two plane mirrors inclined at a given angle: when the mirrors are moved about their line of intersection so as to preserve their inclination, the axes of the pencil after the second reflexion make an angle equal to that which they made after the first reflexion.

11. A circular arc of given length is immersed with its plane vertical, so that the surface of the water bisects its chord at an angle of 45°; find the form of the images of the parts of the arc and chord immersed.

12. A circular arc is placed before a glass sphere at a distance n times the focal length of the sphere from its centre, n being a large number, the axes of the image are as

$$n - \frac{1}{2n} : n.$$

13. An object placed within a spherical drop of water appears $3\frac{1}{2}$ times greater in diameter, than when it is at the principal focus of the sphere.

14. Find the locus of the intersection of the incident and convergent ray of a pencil passing through a sphere, as the pencil is continually diminished.

15. Determine the form of the caustic given by the equation

$$x - y\frac{dx}{dy} = a\frac{dy^2}{dx^2}.$$

16. Find the greatest distance between two lights placed in a straight line parallel to the axis of y, that they may just illuminate the whole curve whose equation is

$$y^2 = \frac{x^3}{4x - 2a};$$

and find the minimum ordinate of the curve.

17. Determine the space left unoccupied between an egg, and a hemispheroidal egg cup in which it stands; the egg being also spheroidal and having an axis-minor equal to that of the cup.

CAIUS COLLEGE, June 1832.

1. The specific gravity of alcohol is 0·796, that of water being unity; find the specific gravity of a mixture of two parts of the former with one of the latter, supposing no condensation to take place.

2. A hemisphere is filled with fluid and inclined at a given angle to the horizon; find how much of the fluid will flow out, and the pressure exerted by the remainder on the surface.

3. The times of oscillation of a pendulum are observed at the Earth's surface and at a given depth below the surface; find from these data the radius of the Earth, supposed spherical.

4. A liquid is contained in a graduated tube; n = the number of divisions at which the liquid stands when the liquid and tube are at a given temperature; n' the number when the temperature is increased by t degrees; shew that the dilatation of the liquid

$$= \frac{n' - n}{n} + \frac{n'\alpha t}{n},$$

where α is the cubical expansion of the material of the vessel for one degree of temperature.

5. Supposing the force which acts on the crank of a steam-engine to be vertical, and to vary as the sine of the angle which the crank makes with the vertical, and the resistance to be uniform; shew that the velocity at the end of a revolution will not be altered, if the moment of the resistance $= \frac{1}{2}$ the greatest moment of the force.

6. If Z be the zenith, P the pole, S the Sun at the beginning of shortest twilight, T the point at which the vertical circle ZS cuts the horizon; prove that $PS + PT = 180°$.

7. A pendulum oscillates in a cycloid on an inclined plane, and the friction on the plane $= n$ times the pressure; shew that the friction will not affect the time of oscillation, and that the pendulum will stop after it has oscillated a number of times $= \dfrac{a - ln \cot . \alpha}{2\, ln \cot . \alpha}$, where a is the original distance from the lowest point.

8. A cube filled with fluid revolves, with a given angular velocity, about a diagonal which is vertical; find the pressure on one of the faces which form the upper angle.

9. A rod falls by gravity in a vertical plane, one extremity sliding on a curve, which is perpendicular to the horizon at the point in which it meets the horizon, and the other sliding along the horizontal plane; shew that the angular velocity of the rod, when it becomes horizontal, is independent of the curve on which one end of it has moved.

10. Two bodies, which are constrained to move in the circumferences of concentric circles, attract each other with forces varying as $\dfrac{1}{(\text{dist.})^2}$; shew that the lengths of the arcs described are inversely as the bodies, and find the time of their small oscillations.

11. The Sun's right ascension at 0^h,

June 3rd, 1832, *in time*, is ...	4^h .	45^m .	$33{\cdot}0^s$.
4th	4 .	49	. 39·5 .
6th	4 .	57	. 53·5 .

Find the Sun's right ascension on the 5th of June, 1832, at 3^h apparent time.

12. A hoop rolls down an inclined plane; compare the velocity with which it leaves the plane, with that with which it begins to roll along the horizontal plane at the foot of the inclined plane.

13. Two cylindrical weights are in equilibrium over a pully when a portion of one of them is immersed in a vessel of fluid; if the surface of the fluid sink uniformly, shew that the weights will have an oscillatory motion relatively to the surface of the fluid; and find its period.

14. The time of describing an elliptic arc

$$= a^{\frac{3}{2}} . (z^1 - z - \sin.z^1 + \sin.z),$$

where

$$z^1 = \cos.^{-1}\left(1 - \frac{r + r^1 + c}{2}\right), \quad z = \cos.^{-1}\left(1 - \frac{r + r^1 - c}{2a}\right).$$

r and r^1 being the radii vectores drawn to the beginning and end of the arc, and c the chord of the arc.

15. Employ the principle of least action, to determine the curve described by a particle acted on by a force tending to a centre and varying as $\frac{1}{(\text{dist.})^2}$.

16. The equation which determines the perturbation in the radius vector of one planet by the disturbing force of another planet, deprived of terms depending on e, is

$$\frac{d^2(r\delta r)}{dt^2} + n^2 . r\delta r + \Sigma. \quad P\cos.(pnt - qn't + Q) = 0,$$

where n is the mean motion of the disturbed, n' that of the disturbing planet. Integrate this equation; and shew

(1). That the eccentricity and place of the perihelion are altered by constant quantities.

(2.) That a force which goes through all its periodic values nearly in the periodic time of the planet, will produce in the radius vector a considerable inequality.

QUEEN'S COLLEGE, 1825.

1. If n be any number whereof the alternate digits beginning with the units are cyphers, and if the sum of the digits be divisible by $r+1$, then the number itself is divisible by $r+1$; r being in the base of the system.

2. The roots of the equation $x^3 - px^2 - px + 1 = 0$, are α, β, and γ; find an expression for $\alpha^m + \beta^m + \gamma^m$.

3. Sum the following series:

$$\frac{1}{1.3.6} + \frac{1}{2.4.7} + \frac{1}{3.5.8} + \text{\&c. to } n \text{ terms.}$$

$$1 + 3 + 6 + 10 + 15 + \text{\&c. to } n \text{ terms.}$$

$$\frac{1}{1.2.3.4} + \frac{3}{2.3.4.5} + \frac{6}{3.4.5.6} + \frac{10}{4.5.6.7} + \text{\&c. to } n \text{ terms.}$$

$$x.\cos.\theta + x^2.\cos.2\theta + x^3.\cos.3\theta, \text{ \&c to } n \text{ terms and to infinity.}$$

4. A given paraboloid standing with its base downwards on a horizontal plane is elevated, so that its base makes with the horizon a given angle; find the position of a given prop which shall support it at that angle, when the pressure on it is a minimum, friction being supposed sufficient to prevent the paraboloid from sliding.

5. Find the centre of gravity of the solid generated by the revolution of the cissoid round its axis.

6. State the principle of virtual velocities, and apply it to find the position of two given weights connected by an inextensible line, and placed in a hemisphere.

7. Find the inclination of a plane, when the time down its length is equal to the time down the length of another inclined plane of the same base, and having its altitude (m) times the altitude of the former.

8. Find in what direction a ball of given elasticity must be projected from a given point in a given circle, so that after (n) reflections at the circumference it may hit a given point within the circle.

9. A straight line of uniform density and thickness, oscillates about a given point in the same plane; find the pressure on the axis in any position, and approximate to the time of an oscillation.

10. A given cylindrical vessel contains equal bulks of three fluids of known specific gravity, which will not mix: divide it into three parts, such that the pressure on each is the same.

11. Find the centre of pressure of a semicircular area cut in the vertical side of a vessel having its diameter inclined at a given angle to the horizon.

12. The force of gravity varying inversely as the n^{th} power of the distance from the centre of the Earth, find the density of the air at any altitude, and shew the method of determining the altitudes of mountains by means of the barometer, on the supposition that the force of gravity is constant: shew also how the result will be affected by the difference of temperature, at the top and bottom of the mountain.

13. Find the time of emptying a semi-paraboloid through a small orifice in its vertex; the axis of the paraboloid being horizontal.

14. Find the time of an oscillation in the arc of a cycloid; the resistance being supposed to vary as the velocity.

15. State the nature of the experiments by which it is proved that light is material.

16. Having given the position of three given straight lines in space, each at right angles to the horizon; find where a spectator must be situated so that they shall appear of the same length.

17. A parabola is immersed vertically in a fluid, so that its base coincides with the surface; find the equation to the image of the curve.

18. Having given the altitudes of the Sun, and of the red ring in the secondary rainbow, it is required to determine the sines of incidence and refraction.

19. In the double concave lens compare the velocities of the foci of incidence and emergence.

20. Shew that there are an infinite number of reflecting curves, which will produce the same caustic, the radiating point being given.

21. Explain the construction and use of the equatoreal instrument.

22. State briefly your reasons for supposing the Earth to be endowed with a motion of rotation round its axis and a motion of translation round the Sun.

23. A known star is observed to have the same altitude as an unknown star: having given the time of the passage of each over the meridian and their common altitude at the time of the first observation, find the latitude of the place.

24. Explain fully the effect of the Sun on the lengths of the lunar months.

25. Explain the nature of centrifugal force, and find the time of describing any portion of the curve in which the centripetal and centrifugal forces are equal to each other.

26. Find the nature of the curve which by its revolution round its axis will generate a surface in which, if the force of gravity act parallel to the axis, the period of a revolution in every section perpendicular to the axis is the same.

QUEEN'S COLLEGE, MAY 1831.

1. A TEA-DEALER mixes together 4lbs. of tea at *5s. 1d.*, 7lbs. at *6s.*, and 9lbs. at *6s. 6d.*: at what rate per lb. must he sell the mixture to gain 5 per cent. by the transaction?

2. Solve the following equations:

(1). $\dfrac{1+\sqrt{(1+4x)}}{\sqrt{(x+\frac{1}{4})}} = 4x.$ (2). $\left.\begin{aligned} 3x^2 &= 2xy + 12 \\ x^4 &= 4x^2y + 144 \end{aligned}\right\}$

(3) $x + y + z = 0,\ (a+b)x + (a+c)y + (b+c)z = 0,$
$abx + acy + bcz = 1.$

3. If a straight line intersect the two sides AC, BC of a plane triangle in the points b, a, and the base AB produced in c; prove that the continued products of their alternate segments thus formed are equal: namely, $Ab \times Bc \times Ca = Ac \times Ba \times Cb$.

4. If any two angles of a triangle be bisected by straight lines, prove that the distance of the point of their intersection from the angle $A = \frac{2bc \cos.\frac{1}{2}A}{a+b+c}$; a, b, c being the sides.

5. Find the polar equation of the circle referred to a given point either within or without the circumference: and thence shew that the rectangle contained by the segments of any chord passing through a fixed point is invariable.

6. A freehold estate is left equally among three persons A, B and C; what numbers of years must A and B successively enjoy it, that the unencumbered reversion may belong to C?

7. A beam of uniform density and thickness fastened at one end by a peg rests upon a wall: it is required to find its inclination to the horizon when the stress upon the peg is m times as great as the pressure upon the wall.

8. If a and b represent the velocities of a body when estimated in directions inclined to each other at an angle α, prove that its velocity estimated in a direction equally inclined to them both is

$$\frac{a+b}{2\cos.\frac{1}{2}\alpha}.$$

9. If the base and vertical angle of a plane triangle be given, prove that the locus of the centres of the inscribed circles is a circle, and find its position and magnitude.

10. Required the number of balls in n courses of a rectangular pile, the length and breadth of the lowest course comprising p and q balls respectively.

11. If X, Y and Z represent the forces acting in the directions of the co-ordinate axes of x, y and z respectively, prove that in the case of equilibrium upon a curved surface $Xdx + Ydy + Zdz = 0$.

12. The vertex of a given cone is attached to the lowest point of a hemispherical bowl: find the stress upon the point of connection when the bowl is filled with fluid and the axis of the cone is in a vertical position: find also the quantity of fluid in the bowl when no stress is exerted there.

13. If any number of straight lines be placed perpendicular to the axis of a spherical reflector, it is required to shew that the image of each will pass through the same two given points.

14. The centre of force being situated in the centre of an equilateral hyperbola, prove that the locus of the points to which a body must move from the curve in the direction of the force to acquire the velocity in the curve is also an equilateral hyperbola, and find its axes.

15. If a and b be the perpendicular altitude and slant side of an upright cone, it is required to shew that the mean distance of the vertex from the base of the cone $= \frac{2}{3}\left(\frac{a^2 + ab + b^2}{a + b}\right)$.

16. If a be any prime number whatever, then will the number of integers less than a^2 and prime to it be expressed by $a(a - 1)$, unity being considered one of them: required a proof.

17. In a spherical triangle whose sides and angles are a, b, c and A, B, C respectively, if

$$\tan.^2\tfrac{1}{2}C = \frac{\cos.\frac{1}{2}(a - b)}{\cos.\frac{1}{2}(a + b)},$$

it is required to shew that $C = A + B$; and from the same expression to prove that if the radius of the sphere be indefinitely increased, the triangle will become right-angled at C.

18. Trace the curve whose equation is $(x^2 + y^2)^2 = a^2x^2 + b^2y^2$: find its points of inflexion and the value of its greatest ordinate.

19. A corpuscle is placed in the axis produced of a given paraboloid of revolution: find the section perpendicular to the axis which attracts it with the greatest force, when the attraction of each particle varies inversely as the square of the distance.

20. Find the latitude of the place where the duration of the twilight is the shortest possible for a given day.

21. Integrate the following differentials:

$$\frac{x^3dx}{(a^2 - x^3)^{\frac{3}{2}}}; \quad \frac{x^4dx}{x^3 + ax^2 - b^2x - ab^2}; \quad e^{a\theta}\sin.^2\theta d\theta:$$

and solve the following differential equations:

(1). $aydx = \{ax - \sqrt{(x^2 \pm y^2)}\}dy.$

(2). $3ydx^2 - 3xdxdy + 4y^2d^2y = 0.$

(3). $5x + y + \frac{dx}{dt} = e^t, \quad 3y - x + \frac{dy}{dt} = e^{2t}.$

22. A perfectly smooth slender rod of given length is perpendicular to the Earth's surface in a given latitude, and a ring of heavy metal descends down it by the force of gravity: required the velocity acquired at the surface and the time of arriving there, the Earth being supposed spheroidal, and to revolve about its axis in 24 hours.

23. The Sun and Moon being situated in the plane of the Earth's equator, shew that the greatest angular separation of high water from the Moon's place $= \frac{1}{2}\sin.^{-1}\frac{S}{M}$, and the corresponding magnitude of the tide $= \sqrt{(M^2 - S^2)}$.

24. Given the length of the axis and the magnitude of the area of the generating curve, to find the form of the vessel of revolution which shall be emptied through a small orifice at the vertex in the least possible time.

QUEEN'S COLLEGE, May 1832.

1. Find the time between twelve and one o'clock at which the hour and minute hands of a watch point exactly in opposite directions.

2. In an obtuse-angled triangle, if perpendiculars be drawn from the angles to the opposite sides, produced if necessary, they will all pass through the same point; required a proof.

3. Solve the following equations:

(1). $\sqrt{(4+x)} + \sqrt{(1+x)} = 2\sqrt{(2+x)}$:

(2). $x^4 - 2x^3 + x - 1 = 0$:

(3). $x + y + z = 6$, $xy + xz + yz = 12$ and $xyz = 8$.

4. Determine the length of a string which shall pass round two wheels of given radii situated in the same plane, and having their centres at a given distance from each other.

5. If the sides and angles of a plane triangle be denoted by a, b, c, A, B, C respectively: then will its area be expressed by

$$\frac{\frac{1}{2^2}(a^2+b^2-c^2)}{\tan.\frac{1}{2}(A+B-C)}$$: required a proof.

6. In the equilibrium upon a curve, it is required to prove that $Xdx + Ydy = 0$: and by means of this equation to compare P and W when they sustain each other upon an inclined plane.

7. Draw a rectilineal asymptote to the conchoid of *Nicomedes*; and if any two radii vectores drawn from the pole C at right angles to each other meet it in Q and R, prove that $\frac{1}{CQ^2} + \frac{1}{CR^2}$ is of invariable magnitude.

8. If from any point K in the focal tangent to an ellipse whose focus is S, a straight line be drawn perpendicular to the axis major at M, and cut the curve in P and Q, then will $KP \cdot KQ = SM^2$; required a proof.

9. If A, B, C be three perfectly elastic balls having the velocities a, b, c, and A impinge upon B, and B upon C, so that their velocities after impact are p, q, r respectively: it is required to shew that

$$Aa^2 + Bb^2 + Cc^2 = Ap^2 + Bq^2 + Cr^2.$$

10. If S and s be the sums due at the ends of the times T and t respectively, and r be the rate of simple interest: prove that the correct equated time of payment is $\frac{ST + st + r(S+s)Tt}{S + s + r(St + sT)}$: and shew whether it is greater or less than that determined by the ordinary rule.

11. If the inclination of a plane be ι, and θ_1, θ_2 be the angles of elevation of two bodies projected with the same velocity from a given point in it: it is required to prove that they will always strike it in the same point when $\theta_1 + \theta_2 = 9 + 0$.

12. A, B, C, a, b, c are the angles and sides of a spherical triangle, and $\tan.^2 \frac{1}{2} C = \frac{\sin. \frac{1}{2}(a - b)}{\sin. \frac{1}{2}(a + b)}$: it is required to prove that $C = A - B$, and to shew how it may be adapted to a plane triangle.

13. Compare the resistance upon a hemispheroid moving in a fluid in the direction of its axis with that upon its circumscribed cylinder.

14. Find where a straight line must be placed perpendicular to the axis of a spherical reflector, that its image may be an ellipse of given eccentricity: and find the corresponding axes and latus rectum of the image.

15. Determine that point in the periphery of an ellipse, having the force in its focus, at which the paracentric velocity is m times as great as the transverse: and prove that no such point can be found if m be greater than $\frac{e}{\sqrt{(1 - e^2)}}$, where e is the eccentricity.

16. Find the conic section to which the equation

$$\sqrt{x} - \sqrt{y} = \sqrt{a}$$

belongs: determine also the positions of its focus and vertex, and the magnitude of its latus rectum.

17. Trace the curve whose equation is $y(x^2 - a^2) = x(x^2 + a^2)$, and determine the natures and positions of all its singular points.

18. Define the centre of percussion: investigate a formula for determining its position, and apply it to the case of a straight rod of uniform density and thickness suspended by a given point.

19. Eliminate by differentiation the exponential and trigonometrical functions from the equation $y = e^x \sec. x$: and transform the equation $y^2 \frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 - y = 0$, so that y shall be the independent variable.

20. In the equation $x^m - a^{m-2}x^2 + b^m = 0$, it is required to prove that there are either one or three possible roots when m is odd, and either four possible roots or none when it is even.

21. In the calculus of finite differences, if $u_x =$ Napierian log.x, then will $\Delta u^x = \frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} - \&c.$: and if $u_x = 1.2.3.\&c.\ x.a^x$, then will $u_{x+1} - a(x+1)u_x = 0$: required a proof.

22. Integrate the following differentials:

$$(1).\ \frac{\left(x^{\frac{2}{3}} + ax^{\frac{1}{3}} + b\right)^{\frac{1}{2}} dx}{x^{\frac{2}{3}}};$$

$$(2).\ (\sin.^{-1}x)^3 dx;$$

$$(3).\ \frac{x^2}{1+x^2}\tan.^{-1}x dx;$$

and find the relations between the variables in the differential equations:

$$(1).\ y^3 + xy^2\frac{dy}{dx} = ax^5;$$

$$(2).\ (x-a)\frac{d^2y}{dx^2} = \left(1 - x\frac{dy}{dx}\right)\frac{dy}{dx}.$$

23. Find the time when a comet appears stationary in its parabolic path, the orbit of the earth being supposed to be a circle in the same plane.

24. Determine the velocity which will be acquired by a globe of given dimensions rolling down the arc of a cycloid, placed with its vertex downwards and its axis vertical.

SIDNEY SUSSEX COLLEGE, May 1829.

1. Extract the square and cube roots of 64·0070202 &c.

2. Find the time between 2 and 3 o'clock, when the hour and minute-hands of a clock make equal angles above and below the line, joining the centre of the face and the 3 o'clock mark.

3. Solve the following equations:

$$(1).\quad \left.\begin{aligned} x + y + z &= a \\ x^2 + y^2 + z^2 &= b^2 \\ x : z &:: x - y : y - z \end{aligned}\right\}$$

$$(2).\quad \frac{x^2 - a^2}{cx} - \frac{cx}{x^2 - a^2} = b$$

4. Let N be any point in the diameter of a circle, whose centre is S, PNQ a chord drawn through N, and join SP: shew geometrically and analytically that PQ is a minimum, and the angle SPQ a maximum, when PQ is perpendicular to the diameter.

5. In a given semi-circle let a second semi-circle be inscribed, in the second a third, and so on for ever: find the point, to which the centre of the inscribed figure is continually approaching as its limit.

6. Compound interest being allowed at a given rate, find the present value of £A, £$2A$, £$3A$, ... £nA, due at the end of 1, 2, 3 ... n years respectively from the present time; and find the value when n is infinite.

7. On the minor axis of a given ellipse, considered as a new major axis, describe an ellipse similar to the given one, and on its minor axis describe another similar ellipse, and so on for ever: find the area of the n^{th} ellipse, and the sum of all the areas to infinity.

8. Let a hemispherical basin of small, but uniform thickness, stand on its vertex on a horizontal plane: if it be loaded at a point in the edge with a given weight, find the position in which it will rest.

9. Find the centre of gravity of the solid formed by the revolution of a given sector of a circle about a radius, which bisects its vertical angle: and find the limits to the place of the centre of gravity, when the angle of the sector is indefinitely diminished, or increased to 180°.

10. Let a cycloid be just immersed perpendicularly in a fluid, so that its surface may be a tangent at the vertex: compare the pressures on the circumscribing rectangle, the cycloid, the isosceles triangle formed by drawing the semi-cycloidal chords, and the generating circle.

11. Find the curve on the horizontal plane, to which a vertical plane reflector must be a constant tangent, so as to make the image of a fixed point in that plane move in a given straight line in the same plane.

12. Let AQ be any arc of a parabola, whose vertex is A, AD a tangent at the vertex, and QD perpendicular to it; also let an indefinite line revolve about the point A, in which let AP be always equal to AD, and the area traced out by AP be always equal to the area AQD: the locus of the point P is a spiral of Archimedes.

13. Find the velocity with which a body must be projected perpendicularly from the Earth's surface, so that it may fall again on the same point after the Earth has made one revolution on its axis; and find the height to which it will rise.

SIDNEY SUSSEX COLLEGE, MAY 1830.

1. If a solid angle be contained by any number of plane angles, their sum is less than four right angles.

2. If P be the product, S the sum, and s the sum of the reciprocals of n quantities in geometrical progression; prove that

$$P^2 = \left(\frac{S}{s}\right)^n$$

3. Solve the following equation:

$$\frac{1+10x}{1-10x} - \frac{1-4x}{1+4x} = 2{\cdot}4$$

4. Let an elastic ball be projected up a given inclined plane with such a velocity as would just carry it to the top: at what point of its ascent must it meet a hard vertical plane, so that, after reflection, it may fall again at the bottom of the given plane?

5. Find the actual time of oscillation in a given hypocycloid.

6. Transform the equation $x^4 + x^3 + x^2 + x + 1 = 0$ into one whose roots shall be the squares of the roots of the given equation: and shew from the roots themselves that the transformation is correct.

7. A table is supported, in a slanting position, on three props, of given but unequal heights, and forming a given triangle on the ground: find its inclination to the horizon.

8. State those cases of oblique spherical triangles, which are sometimes ambiguous: and shew when they are so.

9. Let a body, suspended by a string of given length, be made to describe a conical revolution with a given velocity: find the radius of the circle described, and the periodic time.

10. A body is projected downwards in a medium whose resistance varies as the square of the velocity: determine the circumstances of its descent, according as the initial velocity is greater or less than the terminal velocity.

11. Given the radius of a spherical balloon, the weight of all the apparatus attached to it, and the specific gravity of the gas with which it is filled: find the greatest weight that it can carry up.

12. At a given place on a given day, find the number of hours in the afternoon during which a rainbow can be seen.

13. Let a given triangle be placed with its base on the ground making a given angle with a meridian line, its plane being inclined at a given angle to the horizon: the place, day and hour being known, find the vertical angle of its shadow cast by the Sun on the ground.

14. Explain what is meant by a tautochronous curve: and find it when the force is constant and acts in parallel lines.

15. Investigate fully the general expression for the centre of oscillation of a system of points revolving about a horizontal axis.

16. A body revolving in an ellipse round the focus is reflected towards the centre from any point in the curve between the maximum and mean distances: find the point from which, and the angle at which, it must be again projected, so that its new orbit may be similar and equal to the original one: and compare the time of falling with the periodic time in either orbit.

17. Find the moment of inertia of an ellipse revolving about the centre.

18. Trace the curves, $y^3 = ax^2$, and $y^3 = a^2x$; find the angles which they cut their axis, and the singularity at the point of tersection.

19. Find the integrals of

$$\frac{dx}{x^{\frac{1}{2}}(a^{\frac{1}{2}} + x^{\frac{1}{2}})^2}, \quad \frac{x^n dx}{\sqrt{(2ax - x^2)}} \text{ from } x = 0, \text{ to } x = 2a$$

$$\frac{dx}{(\sin.x)^m . (\cos.x)^n}, \quad m \text{ and } n \text{ being even or odd.}$$

20. Find the accelerating force on a solid cylinder rolling down given inclined plane.

21. Find the attraction of an oblate spheroid on a particle placed its pole.

JESUS COLLEGE, MAY 1830.

1. WHEN three forces act upon a body and keep it at rest, 1) any one of them must be equal and opposite to the resultant of ie other two; (2) and must pass through the intersection of the her two.

2. Define what is meant by "stable and unstable equilibrium" id "equilibrium of indifference," and give an example of each.

3. If any number of forces, in a vertical plane, act upon a body id keep it at rest, the sum of the horizontal and vertical forces is ich = 0.

4. Apply the last stated principle to the solution of the following: weight W is supported by two equal weights P, Q connected by string passing over fixed pullies, A, B in a horizontal line. Find ie position of W.

5. In the parabola, $SP = \frac{2SA}{1 + \cos.ASP}$.

6. Transform the equation $x^4 - a^2x^2 + a^2y^2 = 0$ (1) from ectangular to polar co-ordinates; (2) to an equation between the erpendicular and distance.

7. The roots of $x^3 - qx + r = 0$ are all possible. Shew how they may be found by means of the trigonometrical tables.

8. Two balls A, B of which B is imperfectly elastic, are let fall at the same instant from two points in the same vertical line. Find the point when B, after rebounding from the horizontal plane, will meet A.

9. From what height above a given inclined plane must a perfectly elastic ball descend, so that after impact it may strike a given point upon the plane?

10. Integrate $\dfrac{dx}{x^{2n}\sqrt{(1-x^2)}}$ between $x = 0$, and $x = 1$.

11. In the focal distance SP of a parabola, Sp is taken equal to the ordinate PN. Find the equation to the curve traced out by the point p.

12. Find all the angles in which the curve, whose equation is

$$\left(\frac{x}{y}+\frac{y}{x}\right)^2 = 2a^2.\left(\frac{1}{y^2}-\frac{1}{x^2}\right) \text{ cuts the axis.}$$

13. Determine the maximum ordinate in the equation

$y^3 - axy + x^3 = 0$, and shew that it is a maximum.

14. A pendulum, which vibrates seconds at Greenwich, taken to another place loses n seconds a day. Compare the force of gravity at the two places.

15. If u be a function of x, and in the equation $\dfrac{du}{dx} = 0$ there be m roots equal to a, there will be one minimum value of u corresponding to a if m be odd, and neither maxima nor minima values, if m be even.

16. Draw an asymptote to the spiral, whose equation is

$$\cos.\frac{\theta}{2} = \frac{a}{r}.$$

17. The surface of the solid, generated by the revolution of the cycloid round its base : area of the cycloid :: $8^2 : 3^2$.

JESUS COLLEGE, May 1831.

1. No equation can have more positive roots than changes of signs, nor more negative roots than continuations of the same sign.

2. Find the sum of the m^{th} powers of the roots of an equation, in terms of the sums of the inferior powers, and the coefficients.

3. The accuracy of the approximation to the roots of an equation, depends upon the assumed root being much nearer to one root than to any other.

4. In a parabola the subnormal $= \frac{1}{2}$ latus rectum, and

$$4SP.PV = QV^2.$$

5. In the ellipse $PF.CD = AC.BC$; $SP.HP = CD^2$, and the diameter of curvature $= \dfrac{2CD^2}{PF}$.

6. In the hyperbola; find the equation between its asymptotes.

7. Draw two conjugate diameters inclined at a given angle to each other.

8. Also let QR, qr be two chords intersecting in O; CD, CD' the diameters parallel to them, then

$$QO.OR : qO.Or :: CD^2 : CD'^2.$$

9. Find the equation to the section of a right cone made by any plane.

10. If $y = e^x \sec.x$: then $\frac{1}{2}\left(y + \frac{dy}{dx}\right) y.\frac{d^2y}{dx^2} = y^3 + \left(\frac{dy}{dx}\right)^3$.

11. Find the equation to the curve in which the perpendicular on the tangent from the foot of the ordinate is constant.

12. Find the locus of the middle points of all equal chords in an ellipse.

13. A cycloid rolls upon an equal one, find the locus of the vertex of the rolling cycloid.

14. APQ is an indefinite straight line given in position; two lines SP, SQ are drawn from a fixed point S, and PO, QO inclined at a constant angle to SP, and SQ; it is required to find the locus of the ultimate intersections of PO, and QO.

15. Find the magnitude and position of the resultant of any number of parallel forces.

16. The times of descent down all chords of a vertical circle drawn from the highest or lowest points are equal.

17. Find the direction in which a body must be projected with a given velocity to hit a given mark.

JESUS COLLEGE, May 1831.

1. The greater side of every triangle is opposite to the greater angle.

2. In a circle, the angle in a semicircle is a right angle.

3. Similar triangles are to one another in the duplicate ratio of their homologous sides.

4. If two straight lines be at right angles to the same plane, they shall be parallel to one another.

5. Find the value of $\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$ to three places of decimals.

6. Explain the method of transforming a number from one scale of notation to another. Apply the rule to the case of decimals; and transform 13·454 from a system in which the radix is 8, to another in which the radix is 4.

7. Write down the expansion of $(1-x)^{-\frac{1}{2}}$ by the Binomial Theorem, and give the general term.

8. Insert n geometric means between a and b, and sum the series $\frac{x}{\sqrt{y}}+\sqrt{x}+\sqrt{y}+\frac{y}{\sqrt{x}}+$ &c. to n terms and to infinity.

9. Prove that

$$1-2n+3.n.\frac{n+1}{2}-4.n.\frac{n+1}{2}.\frac{n+2}{3}+\&c.=\frac{2-n}{2^{n+1}}.$$

10. When the square root of $a+\sqrt{b}+\sqrt{c}+\sqrt{d}$ can be reduced to the form $\sqrt{\alpha}+\sqrt{\beta}+\sqrt{\gamma}$; then

$$2a\sqrt{(bcd)}=bc+bd+cd.$$

Apply this criterion to shew that the square root of

$$6+\sqrt{8}+\sqrt{12}+\sqrt{24}$$

can be extracted in the above form; and find the root.

11. Find the present value of an annuity to continue for n years, allowing compound interest. What is the present value of a perpetual annuity?

12. If α and β, when substituted for x and y, satisfy the equation $ax + by = c$ where a and b are prime to each other; then the number of corresponding positive integer values of x and y will be $1 + l\frac{\beta}{a} + l\frac{\alpha}{b}$: where $l\frac{\beta}{a}$, $l\frac{\alpha}{b}$ denote the least whole numbers less than $\frac{\beta}{a}$ and $\frac{\alpha}{b}$.

13. Also, if two corresponding values of x and y in the equation

$$3079x + 2711y = 37819000 \text{ are } 7000 \text{ and } 6000.$$

Find all their positive integer values.

14. Shew that $a^x = 1 + px + \frac{p^2x^2}{1.2} + \&c.$ where

$$p = 2\left\{\frac{a-1}{a+1} + \frac{1}{3}\left(\frac{a-1}{a+1}\right)^3 + \&c.\right\}$$

What is meant by a Naperian logarithm? Find the value of its base; and shew that $p =$ Nap. log.a.

15. If $\log._a x$ denote the logarithm of x to the base a: then

$$\log._b x = \frac{\log._a x}{\log._a b}.$$

16. Find the number corresponding to a logarithm, not exactly found in the tables.

17. $\log._e(n+1) = 2\log._e n - \log._e(n-1)$

$$-2\left\{\frac{1}{2n^2+1} + \frac{1}{3}\cdot\frac{1}{(2n^2+1)^3} + \frac{1}{5}\frac{1}{(2n^2+1)^5}\&c.\right\}$$

THE END.

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